Definition 1 *A* clique *in an undirected graph $G = (V, E)$ is a subset of the vertex set $C \subset V$, such that for every two vertices in C, there exists an edge connecting the two. This is equivalent to saying that the subgraph induced by C is complete (in some cases, the term clique may also refer to the subgraph).*

Definition 2 *The clique polynomial of a graph G, denoted by C(G;x), is defined by*

$$C(G;x) = \sum_{k=0}^{n} a_k(G)x^k,$$

where $a_0(G)=1$ and $a_k(G)$ is the number of k-cliques of G. For $V(G)=\emptyset$ we define C(G;x)=1.

Definition 3 *Considere un nmero natural n. Definimos el* grafo de la fraccin n de un punto *mediante el binomio 1+x/n. Denotamos tal grafo por $K_{1/n}$.*

1

Definition 4 *The* lexicographic product *or* graph composition $G \bullet H$ *of graphs G and H is a graph such that*

- *the vertex set of $G \bullet H$ is the cartesian product $V(G) \times V(H)$; and*
- *any two vertices (u,v) and (x,y) are adjacent in $G \bullet H$ if and only if either u is adjacent with x in G or $u = x$ and v is adjacent with y in H.*

Lemma 1 *El polinomio de camarillas de la composicin $G \bullet H$ es igual a la composicin siguiente.*

$$C(G \bullet H; x) = C(G; C(H; x) - 1).$$

Proof: Este lema es una consecuencia de Theorem 1 in [1], Definition 2.2 in [5] and Fact 12 in [3], page 489. □

Denotamos por K_1 el grafo que consiste de un punto.

Considere un nmero natural n. Escribamos los polinomios de camarillas siguientes.

$$\begin{aligned} C(K_1; x) &= 1 + x, \\ C(K_1 \cup \cdots (n \text{ times}) \cdots \cup K_1; x) &= 1 + nx. \end{aligned}$$

Denotamos por

$$C(K_{1/n}; x) = 1 + x/n.$$

Tenemos la igualdad siguiente.

$$C(K_1; x) = C(K_{1/n}; C(K_1 \cup \cdots (n \text{ times}) \cdots \cup K_1; x) - 1).$$

En vista de Lema 1, podemos interpretar la definicin del grafo $K_{1/n}$ como un *grafo* que al componerlo con n puntos resulta un punto.

2

Definition 5 *The* graph join *of two graphs, G+H, is their graph union with all the edges that connect the vertices of the first graph with the vertices of the second graph [4].*

Citamos Teorema 2.3(b) en [5].

Theorem 1 *Let G_1 and G_2 be two vertex-disjoint graphs. Then*

$$C(G_1 + G_2; x) = C(G_1; x) \cdot C(G_2; x).$$

En vista de Teorema 1, definimos lo siguiente.

Definition 6 *Considere un nmero natural n. Definimos la* juntura $K_{1/n}$*+$\cdots$(n times)$\cdots$+$K_{1/n}$ mediante el polinomio (1+x/n)^n.*

Escribamos el lmite siguiente.

$$\lim_{n\to\infty} \left(1 + \frac{x}{n}\right)^n = \exp(x). \tag{1}$$

En vista de Definicin 6 y Lmite 1, definimos lo siguiente.

Definition 7 *Definimos el* punto exponente *exp(p) mediante la funcin exp(x).*

3

Definition 8 *An* independent set *or* stable set *is a set of vertices in a graph, no two of which are adjacent. That is, it is a set I of vertices such that for every two vertices in I, there is no edge connecting the two. Equivalently, each edge in the graph has at most one endpoint in I. The size of an independent set is the number of vertices it contains.*

The independence number *$\alpha(G)$ of a graph G is the size of the largest independent set of G.*

The independence number of a lexicographic product may be easily calculated from that of its factors [2]:

$$\alpha(G \bullet H) = \alpha(G)\alpha(H). \tag{2}$$

Segn nuestra interpretacin del grafo $K_{1/n}$, y en vista de Frmula 2, definimos lo siguiente.

Definition 9 *Considere un nmero natural n. El nmero de independencia de la fraccin n de un punto es 1/n.*

Lemma 2 *Considere dos grafos G y H. El nmero de independencia de la juntura G+H es el mayor de los nmeros de independencia de los grafos G y H.*

Proof: Este lema es una consecuencia de Teorema 2.3(b) en [5]. □

En vista de las definiciones 6, 9 y Lema 2, definimos lo siguiente.

Definition 10 *Considere un nmero natural n. El nmero de independencia de la juntura $K_{1/n}+\cdots$(n times)$\cdots+K_{1/n}$ es 1/n.*

En vista de las definiciones 7 y 10, definimos lo siguiente.

Definition 11 *El nmero de independencia del punto exponente es cero.*

4

Definition 12 *The* clique number *$\omega(G)$ of a graph G is the number of vertices in a maximum clique in G.*

As the independence number of a lexicographic product, the clique number of a lexicographic product is as well multiplicative [2]:

$$\omega(G \bullet H) = \omega(G)\omega(H). \tag{3}$$

Segn nuestra interpretacin del grafo $K_{1/n}$, y en vista de Frmula 3, definimos lo siguiente.

Definition 13 *Considere un nmero natural n. El nmero de camarilla de la fraccin n de un punto es 1.*

Lemma 3 *Considere dos grafos G y H. El nmero de camarilla de la juntura $G+H$ es la suma de los nmeros de camarilla de los grafos G y H.*

Proof: Este lema es una consecuencia de Teorema 2.3(b) en [5]. □

En vista de las definiciones 6, 13 y Lema 3, definimos lo siguiente.

Definition 14 *Considere un nmero natural n. El nmero de camarilla de la juntura $K_{1/n}+\cdots$(n times)$\cdots+K_{1/n}$ es n.*

En vista de las definiciones 7 y 14, definimos lo siguiente.

Definition 15 *El nmero de camarilla del punto exponente es ∞.*

5

Definition 16 *A* coloring *of a graph is a labelling of the graph's vertices with colors such that no two vertices sharing the same edge have the same color.*

The smallest number of colors needed to color a graph G is called its chromatic number*, and is denoted $\chi(G)$.*

La desigualdad $\omega(G)\leq \chi(G)$ aparece en la pgina 432 de [3]. En vista de esta desigualdad, definimos lo siguiente.

Definition 17 *El nmero cromtico del punto exponente es ∞.*

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