On the Hessian topology of real algebraic surfaces

Adriana Ortiz

In this talk we give some results about the Hessian topology of real algebraic surfaces which are the graph of some real polynomial function in two variables. In particular, an index formula for the field of asymptotic directions involving the number of connected components of the Hessian curve constituting the boundary of the unbounded connected component of the complement of the Hessian curve, and the number of the corresponding Gaussian cusps will be given.

Second order geometry of spacelike surfaces in de Sitter 5-space

Ana Claudia Nabarro (joint work with M. Kasedo and M.A.S. Ruas)

De Sitter space is known as a Lorentz space with a positive constant curvature in the Minkowski space. A Surface with a Riemannian metric is called a spacelike surface. The aim of this work is to investigate geometrical properties on the spacelike surfaces in the de Sitter space, specially the properties of the curvature ellipse in the case of spacelike surfaces in de Sitter space S_1^5 by using the actions $SO(1,2)$ and $GL(2,\mathbb{R})$ on the system of conics defined by the second fundamental form. The main results are the classification of the second fundamental mapping and the description of the possible configurations of the LMN-ellipse. This ellipse gives informations on the binormal directions and consequently about asymptotic directions.

The degeneration of the boundary of the Milnor fibre to the link of complex and real non-isolated singularities

Aurelio Menegon Neto

joint work with José Seade

Let $f : (\mathbf{R}^m, 0) \to (\mathbf{R}^k, 0)$, $m \geq k$, be a real analytic singularity with an isolated critical value and the Thom af-property. We will define the vanishing zone for f and we will give necessary and sufficient conditions for it to be a fibre bundle over the link of the singular set of $V(f)$. Then we will describe the degeneration of the boundary of the Milnor fibre to the link of some families of real analytic singularities using L-'s polyhedra.

Stratification of the space of foliations on $\mathbb{CP}^2$

Claudia, Reynoso Alcántara
Universidad de Guanajuato, México

In this talk we will construct a stratification of the space of foliations on $\mathbb{CP}^2$ of degree d with respect to the action by change of coordinates of $Aut(\mathbb{CP}^2)$. We use the Norbert A'Campo's implementation of Popov's algorithm to obtain the indexing set of the stratification and the dimension of the strata. In some cases we characterize the foliations on every stratum according to existence of degenerate singular points and algebraic leaves. These strata are non-singular, locally-closed, algebraic varieties. The aim of the talk is to give arguments to convince us of the usefulness of this stratification to classify foliations with special properties.

Zeta Functions of Laurent Polynomials

Edwin León Cardenal

This talk is about singularities over $p-$adic fields, a topic which has received a lot of attention lately. Here we study local zeta functions attached to Laurent polynomials over $p-$adic fields, with the condition of being non-degenerate with respect to their Newton polytopes at infinity. As an application we obtain asymptotic expansions for $p-$adic oscillatory integrals attached to Laurent polynomials. We show the existence of two different asymptotic expansions for such integrals, one when the absolute value of the parameter approaches infinity, the other when the absolute value of the parameter approaches zero. These two asymptotic expansions are controlled by the poles of twisted local zeta functions of Igusa type. This is a joint work with W. A. Zúñiga-Galindo.

The topology of real suspension singularities of type $f\bar{g} + z^n$

Haydée Aguilar-Cabrera

In this talk we present some results on the topology of the family of real analytic germs $F\colon (\mathbb{C}^3, 0) \to (\mathbb{C}, 0)$ with isolated critical point at 0, given by $F(x, y, z) = f(x, y)\overline{g(x, y)} + z^r$, where f and g are holomorphic, $r \in \mathbb{Z}^+$ and $r \geq 2$. We describe the link L_F as a graph manifold using its natural open book decomposition, related to the Milnor fibration of the map-germ $f\bar{g}$ and the description of its monodromy as a quasi-periodic diffeomorphism through its Nielsen invariants. Furthermore, such a germ F gives rise to a Milnor fibration $\frac{F}{|F|}\colon \mathbb{S}^5 \setminus L_F \to \mathbb{S}^1$. We present a join theorem, which allows us to describe the homotopy type of the Milnor fibre of F and we show some cases where the open book decomposition of $\mathbb{S}^5$ given by the Milnor fibration of F cannot come from the Milnor fibration of a complex singularity in $\mathbb{C}^3$.

Collapsing topology at singular points

Lev Birbrair

We study isolated singular points of real and complex algebraic and semialgebraic sets from a metric viewpoint. The topology of the link of the singular set is collapsing at the singular point. For the metric study it is important to see a difference between slow collapsing topology and fast collapsing topology. We show that there exists a so-called thin-thick decomposition near singular points. The thick part is responsible to the slow collapsing topology and the fast collapsing topology collapses only at the thin part. We show that this decomposition is canonical with respect to some natural equvalence relation. We are also going to make some conjectures, relating the theory with Lipschitz Geometry. Based on joint works with Alexandre Fernandes, Vincent Grandjean and Donal O'Shea.

Computing Milnor classes using Lê cycles

Michelle F. Z. Morgado
Instituto de Biociências Letras e Ciências Exatas,
Universidade Estadual Paulista-UNESP,
email: mmorgado@ibilce.unesp.br

Joint work with Roberto Callejas-Bedregal and José Seade.

Abstract

The main goal of this talk is to present a link between the theory of Chern classes for singular varieties and the geometry of the varieties in question. Namely, we show that if Z is a hypersurface in a compact complex manifold, defined by the zero-scheme of a nonzero holomorphic section of a very ample line bundle, then its Milnor classes, regarded as elements in the Chow group of Z, are determined by the global Lê cycles of Z. Morally this implies, among other things, that the Milnor classes determine the topology of the local Milnor fibres at each point of Z, and the geometry of the local Milnor fibres determines the corresponding Milnor classes.

Algebraic methods applied to mappings under relative symmetries

Miriam Manoel

June 20, 2013

Symmetries of a mapping form a group with a linear real action on the space of variables that commutes with the mapping. Relative symmetries extend this notion, forming a larger group Γ whose subset of symmetries is a normal subgroup H of finite index. The actions of Γ on source and target are now distinct. More precisely, consider the cyclic group $\mathbf{Z}_m$, where m is the index of H, and an epimorphism $\sigma : \Gamma \to \mathbf{Z}_m$; for $\gamma \in \Gamma$, if $x \mapsto \gamma x$ denotes the action on the source, then $x \mapsto \sigma(\gamma)\gamma x$ is the action on the target. In this talk, we discuss the power of algebraic tools from invariant theory to obtain the general form of mappings with relative symmetries, as well as for the systematic study of the singularities and bifurcations of vector fields under relative symmetries. In particular, we shall address more attention to $m = 2$, which is the case of equivariant and reversible vector fields. The results presented here are part of a series of joint works with P.H. Baptistelli.

On foliations by curves in the projective plane having a very special subscheme of its singularities.

Jorge Olivares

Abstract

Let $\mathbb{P}^2$ denote the complex projective plane and $\Omega^1_{\mathbb{P}^2}$ and $\Theta_{\mathbb{P}^2}$ its cotangent and tangent sheaves, respectively. A foliation by curves with singularities (a *foliation* in the sequel) of degree r on $\mathbb{P}^2$ is the class

$$\mathcal{F} = [\Omega] \in \mathbb{P}\mathrm{H}^0(\mathbb{P}^2, \Omega^1_{\mathbb{P}^2}(r+2)) \text{ (or } [\alpha] \in \mathbb{P}\mathrm{H}^0(\mathbb{P}^2, \Theta_{\mathbb{P}^2}(r-1)))$$

of a global section $\Omega \in \mathrm{H}^0(\mathbb{P}^2, \Omega^1_{\mathbb{P}^2}(r+2))$. In affine coordinates (X_0, X_1, X_2) of $\mathbb{C}^3$, the section Ω corresponds to a $1-$form $\Omega = \sum_{i=0}^{2} A_i dX_i$, where A_i are homogeneous polynomials of degree $r+1$ satisfying the *Euler condition* $\sum_{i=0}^{2} X_i A_i = 0$.

The *singular scheme* $\mathrm{S}(\mathcal{F})$ of $\mathcal{F}$ is the scheme of zeroes of a section $\Omega \in \mathcal{F}$ and we say that $\mathcal{F}$ has *isolated singularities* if $\mathrm{S}(\mathcal{F})$ is zero-dimensional.

A foliation with isolated singularities is determined by its singular scheme in the following sense: Let $\mathcal{F}$ and $\mathcal{F}'$ be two foliations of degree $r \geq 2$, on $\mathbb{P}^2$. If $\mathcal{F}$ has isolated singularities and $\mathrm{S}(\mathcal{F}') \supseteq \mathrm{S}(\mathcal{F})$, then $\mathcal{F}' = \mathcal{F}$ (see [1]).

Assume that $\mathcal{F}$ has isolated singularities. If moreover $\mathrm{S}(\mathcal{F})$ is reduced, then there exist proper subschemes $Z \subset \mathrm{S}(\mathcal{F})$ which still determine $\mathcal{F}$ in the sense above: if $\mathrm{S}(\mathcal{F}') \supseteq Z$, then $\mathcal{F}' = \mathcal{F}$. Such subschemes were called *special* in [2].

A *very special* subscheme $\hat{Z} \subset \mathrm{S}(\mathcal{F})$ is one that determines $\mathcal{F}$ in the sense above and whose degree $\deg\hat{Z}$ is minimal with respect to this property.

In the talk we will compute the minimal degree $\mu(2, r-1)$ from above and will prove that the set of foliations of degree r whose singular scheme contains a very special subscheme contains a non-empty open set.

Most of this work is join with A. Campillo.

1. A. Campillo, J. Olivares. *On sections with isolated singularities of twisted bundles and applications to foliations by curves*, Math. Res. Lett., 10, 2003, 651-658.

2. A. Campillo, J. Olivares. *Special subschemes of the scheme of singularities of a plane foliation*, C. R. Math. Acad. Sci. Paris, 344, 2007, 9, 581-585.

ENRIQUES DIAGRAMS AND EQUISINGULAR STRATA OF FAMILIES OF CURVES

RAGNI PIENE

Abstract. Given a singular point on a curve lying on a smooth surface one associates a weighted sequence of infinitely near points, combinatorially expressed by an Enriques diagram. The singularity defines a fat point on the surface, and the Enriques diagram can be recovered also from the fat point. For a given Enriques diagram and a given surface, the set of fat points with this diagram forms a smooth subscheme of the Hilbert scheme of the surface. The dimension of the subscheme is expressed in terms of the numerical invariants of the diagram, and I will relate the latter to the numerical invariants of the singularity. I will briefly mention how this theory can be applied to study curves in a family with singularities of prescribed topological type.

This is joint work with Steven Kleiman.

CMA/Department of Mathematics, University of Oslo, PO Box 1053, Blindern, NO-0316 Oslo, Norway

E-mail address: ragnip@math.uio.no

A note on the Mond conjecture

Raul Oset

The classification of multigerms under A-equivalence has had a new impulse substituting the classical classification techniques by operations in order to obtain multigerms from germs and multigerms in lower dimension and codimension. Examples of these operations are augmentations and concatenations. Oset Sinha, Ruas and Wik-Atique recently defined the operation of simultaneous augmentation and monic concatenation.

On the other hand, an open problem related to the classification of germs is the Mond conjecture which relates an algebraic invariant of a germ (the A_e-codimension) with the topology of a stablisation of it (its image Milnor number).

We will prove the Mond conjecture for mulitgerms resulting from the operation of simultaneous augmentation and monic concatenation.

Joint work with Catiana Casonatto.

Global phase portraits of quadratic systems with a semi--elemental triple node

Regilene Oliveira

Planar quadratic differential systems occur in many areas of applied mathematics. Although more than one thousand papers have been written on these systems, a complete understanding of this family is still missing. Classical problems, and in particular, Hilbert's 16th problem, are still open for this family. In this talk we shall discuss how to make a global study of the family $QT\overline{N}$ of all real quadratic polynomial differential systems which have a semi--elemental triple node (triple node with exactly one zero eigenvalue). This family modulo the action of the affine group and time homotheties is three--dimensional and we give its bifurcation diagram with respect to a normal form, in the three--dimensional real space of the parameters of this form. This bifurcation diagram yields 28 phase portraits for systems in $QT\overline{N}$ counting phase portraits with and without limit cycles. Algebraic invariants are used to construct the bifurcation set. The phase portraits are represented on the Poincaré disk. The bifurcation set is not only algebraic due to the presence of a surface found numerically.

This is a joint work with Alex C. Rezende (ICMC-USP) and Joan C. Artes (UAB-Barcelona).

On the classification of generic germs of foliations in the complex plane

Ernesto Rosales

In a joint work with L.Ortiz-Bobadilla y S.Voronin, we consider the class of germs of holomorphic vector fields in $(C^2, 0)$ with vanishing n-jet at the origin, n>1 and the family of foliations generated by them. For generic germs we prove the existence of formal normal forms wich classify the formal and the analytic class of the foliations. These formal normal forms in the least degenerated case are analytical. For this analysis we consider the dicritic case (with infinite number of separatrices: see [1], [2]) and the nondicritic case (with a finite number of separatrices: see [3], [4]).

[1]. Ortiz-Bobadilla,L. Rosales-Gonzalez,E. Voronin, S.M., *Rigidity theorems for generic holomorphic germs of dicritic foliations and vector fields in $(C^2,0)$*, Mosc. Math. J. 5 (2005), no. 1, 171--206.

[2]. Ortiz-Bobadilla,L.; Rosales-Gonzalez,E.; Voronin, S., *Analytic normal forms of germs of holomorphic dicritic foliations.*, Moscow Math. Journal, V8, N3, pp 1-25 (2008).

[3]. Ortiz-Bobadilla L, Rosales Gonzalez E, Voronin S., Thom's problem for degenerated singular points of holomorphic foliations in the plane, MMJ,Moscow Mathematical Journal,internacional, 2012, V12, N4, pp 825-862

[4]. Ortiz-Bobadilla L., Rosales-González E., Voronin,S.M; *Formal and Analytic normal forms of germs of holomorphic nondicritic foliations.*

Topological classification and stems of co-rank two map germs from the plane to the plane.

Marcelo J. Saia
ICMC-USP. São Carlos.

Joint work with: Aldicio J. Miranda and Liane M. F. Soares.

Abstract

The main purpose of this work is to describe the topological orbits which are in a $\mathcal{K}$-orbit of finitely determined map germs from $\mathbb{C}^2$ to $\mathbb{C}^2$ to. At least for a very large number of co-rank two $\mathcal{K}$-orbits. The co-rank one case is described by Gaffney and Mond in [1, Proposition 4.5. and Theorem 4.6].

The simplest case of $\mathcal{A}$-finitely determined co-rank two map germs is the $\mathcal{K}$-class $\mathcal{K}(xy, x^2 + y^3)$, Gaffney and Mond in [1, Example 5.11] showed that there exists only one topological orbit in this $\mathcal{K}$-class. The next example given by Gaffney-Mond in [1, Example 5.12] is the $\mathcal{K}$-class $\mathcal{K}(xy, x^3 + y^4)$, in this case they expected that the number of topological orbits was finite.

To our surprise, we found a special type of germs in this $\mathcal{K}$-class that are not $\mathcal{A}$-finitely determined, moreover from these germs we showed that there exists a non finite number of $\mathcal{A}$-finitely determined germs in this $\mathcal{K}$-class which belong to different topological orbits. These special germs are called *stems* by D. Mond in [2] and are well known in the class of germs of maps from surfaces to 3-space, the germs S_∞, B_∞ and H_∞.

Therefore, the most natural step in this work was to search for stems in other $\mathcal{K}$-classes. Following this point of view we give a complete answer for this question for any given co-rank two $\mathcal{K}$-class with finitely determined normal form. We show how to obtain stems in any $\mathcal{K}$ class $(xy, x^a + y^b)$, the only exceptions are the cases $(2, 3)$ and $(2, 5)$.

Moreover, from the stems in these $\mathcal{K}$-classes we can show that there exists a non finite number of topological orbits. Again the the only exceptions are the cases $(2, 3)$ with one topological orbit and $(2, 5)$ with two topological orbits.

References

[1] Gaffney T. and Mond D., *Weighted homogeneous maps from the plane to the plane*. Math. Proc. Camb. Phil. Soc., vol. 109, 451–470, 1991.

[2] Mond D. *Some Remarks on the Geometry and classification of germs of maps from surfaces to* 3*-space*. Topology **26**, 3, 1987, 361–383.

ASYMPTOTIC LINES OF SURFACES IN $\mathbb{R}^n$, $n = 4, 5$

FEDERICO SÁNCHEZ-BRINGAS

Abstract. We describe the differential equations of the asymptotic lines on a surface in $\mathbb{R}^4$ in terms of some new invariants, by means of the Gauss map, and analyze some of their geometric properties. Moreover, we consider a natural generalization of this approach for surfaces in $\mathbb{R}^5$. Joint work with Pierre Bayard.

SINGULARITIES OF QUADRATIC MAPPINGS.

Santiago López de Medrano.
Joint work with Y. Barreto, S. Gitler, V. Gómez and A. Verjovsky.

Abstract: We study homogeneous mappings of degree 2 $F : \mathbf{R}^n \to \mathbf{R}^k$, especially the topology of the variety $F^{-1}(0)$ and its intersection Y with the unit sphere, and some other related varieties. These functions appear frequently (and provide interesting examples) in various areas of mathematics, being the 2-jets of functions at points of rank 0 and despite the fact that for $k > 1$ they are quite unstable. This study was started decades ago by C.T.C. Wall and by the author, in the case $k = 2$, but many questions were left open.

In this talk we will present a survey of recent results about these varieties. They include:

I. The description of the topology of Y in the generic case for $k = 2$: Y is diffeomorphic to one of the following:
a) The unit tangent bundle of a sphere, or
b) The product of two spheres, or
c) The product of three spheres, or
d) The connected sum of an odd number of manifolds, each of them a product of two spheres.

II. The description (with some restrictions) of related varieties in the case $k = 2$, such as other fibers of F, semi-algebraic varieties given by adding inequalities, open book decompositions, etc.

III. Results for the case $k > 2$, necessarily partial, but which give infinite families of functions for which the topology of Y is of the type found for $k = 2$, their products or other simple constructions on them.

[1] C.T.C. Wall, Stability, pencils and polytopes, Bull. London Math. Soc. 12 (1980), 401421.
[2] S. Lpez de Medrano, Topology of the intersection of quadrics in R^n, Lecture Notes in Mathematics 1370 (1989), 280292.
[3] S. Gitler and S. López de Medrano, *Intersections of Quadrics, Moment-Angle Manifolds and Connected Sums*, axXiv:0901v2, 2009, Geometry and Topology 2013.

[4] S. López de Medrano, *Singularities of real homogeneous quadratic mappings*, RACSAM, Series A., doi:10.1007/s13398-012-0102-6, 2012.

[5] Y. Barreto, S. López de Medrano and A. Verjovsky, *Moment-angle manifolds, intersection of quadrics and higher dimensional contact manifolds*, arXiv 1305.3208.

[6] V. Gómez Gutiérrez and S. López de Medrano, *Topology of the intersection of quadrics in R^n II*, to appear.

Oscillatory Integrals for Meromorphic Functions

W. A. Zúñiga Galindo
Centro de Investigación y de Estudios Avanzados del Instituto Politécnico Nacional
México

We attach to a germ of meromorphic function f, real or complex, an oscillatory integral depending on a parameter. We prove the existence of two different asymptotic expansions for this type of integrals. One, when the norm of the parameter tends to infinity, and other when the norm of the parameter tends to zero. Each of these expansions involve functions of type $A\left(\frac{z}{|z|}\right)|z|^N \ln^v |z|$ where the pairs (N, v) are obtained from an embedded resolution of singularities of a divisor determined by f. We follow a classical approach, which consists in, first, establishing the meromorphic continuation of the local zeta functions attached to f, and then in showing that the asymptotic expansions for the corresponding oscillatory integrals are controlled by the poles of the local zeta functions.

The announced results form part of an ongoing work of the author with Willem Veys.

.

A Geometric Interpretation of Grothendieck Duality in the Jacobian Algebra

Xavier Gómez-Mont

CIMAT

Let $f : \mathbb{C}^{n+1} \longrightarrow \mathbb{C}$ be a germ of a holomorphic function with an isolated singularity and

$$\mathbb{A} := \frac{\mathcal{O}_{\mathbb{C}^{n+1},0}}{(\frac{\partial f}{z_0}, \ldots, \frac{\partial f}{z_n})}$$

the Jacobian algebra of f, which is a $\mathbb{C}$-finite dimensional vector space, which comes provided with a distinguished element, the class of the Hessian of f, $Hess(f)$, (it generates the 1 dimensional socle). The multiplicative structure on $\mathbb{A}$:

$$\mathbb{A} \times \mathbb{A} \longrightarrow \mathbb{A}$$

together with Grothendieck residue map

$$L : \mathbb{A} \longrightarrow \mathbb{C} \qquad\qquad L(g) := \int_{|\frac{\partial f}{z_0}|=\cdots=|\frac{\partial f}{z_n}|=\varepsilon} \frac{g(z_0, \ldots, z_n) dz_0 \wedge \ldots \wedge dz_n}{\frac{\partial f}{z_0} \cdots \frac{\partial f}{z_n}}$$

gives Grothendieck non-degenerate bilinear form in the Milnor Algebra

$$\mathbb{A} \times \mathbb{A} \longrightarrow \mathbb{C} \qquad\qquad < g, h >:= L(gh).$$

We will describe a geometric/topological interpretation of this bilinear form in terms of the of the cup product of the vanishing cycles at the singularity. We will show that the bilinear form may be decomposed into simpler primitive forms using the endomorphism of $\mathbb{A}$ obtained by multiplication with f.