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A Geometric Interpretation of Grothendieck Duality in the Jacobian Algebra

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Let $f : \mathbb{C}^{n+1} \longrightarrow \mathbb{C}$ be a germ of a holomorphic function with an isolated singularity and

$$\mathbb{A} := \frac{\mathcal{O}_{\mathbb{C}^{n+1},0}}{(\frac{\partial f}{z_0}, \ldots, \frac{\partial f}{z_n})}$$

the Jacobian algebra of f, which is a $\mathbb{C}$-finite dimensional vector space, which comes provided with a distinguished element, the class of the Hessian of f, $Hess(f)$, (it generates the 1 dimensional socle). The multiplicative structure on $\mathbb{A}$:

$$\mathbb{A} \times \mathbb{A} \longrightarrow \mathbb{A}$$

together with Grothendieck residue map

$$L : \mathbb{A} \longrightarrow \mathbb{C} \qquad\qquad L(g) := \int_{|\frac{\partial f}{z_0}| = \cdots = |\frac{\partial f}{z_n}| = \varepsilon} \frac{g(z_0, \ldots, z_n) dz_0 \wedge \ldots \wedge dz_n}{\frac{\partial f}{z_0} \cdots \frac{\partial f}{z_n}}$$

gives Grothendieck non-degenerate bilinear form in the Milnor Algebra

$$\mathbb{A} \times \mathbb{A} \longrightarrow \mathbb{C} \qquad\qquad < g, h > := L(gh).$$

We will describe a geometric/topological interpretation of this bilinear form in terms of the of the cup product of the vanishing cycles at the singularity. We will show that the bilinear form may be decomposed into simpler primitive forms using the endomorphism of $\mathbb{A}$ obtained by multiplication with f.