SINGULARITIES OF QUADRATIC MAPPINGS.

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Abstract: We study homogeneous mappings of degree 2 $F : \mathbf{R}^n \to \mathbf{R}^k$, especially the topology of the variety $F^{-1}(0)$ and its intersection Y with the unit sphere, and some other related varieties. These functions appear frequently (and provide interesting examples) in various areas of mathematics, being the 2-jets of functions at points of rank 0 and despite the fact that for $k > 1$ they are quite unstable. This study was started decades ago by C.T.C. Wall and by the author, in the case $k = 2$, but many questions were left open.

In this talk we will present a survey of recent results about these varieties. They include:

I. The description of the topology of Y in the generic case for $k = 2$: Y is diffeomorphic to one of the following:
a) The unit tangent bundle of a sphere, or
b) The product of two spheres, or
c) The product of three spheres, or
d) The connected sum of an odd number of manifolds, each of them a product of two spheres.

II. The description (with some restrictions) of related varieties in the case $k = 2$, such as other fibers of F, semi-algebraic varieties given by adding inequalities, open book decompositions, etc.

III. Results for the case $k > 2$, necessarily partial, but which give infinite families of functions for which the topology of Y is of the type found for $k = 2$, their products or other simple constructions on them.

[1] C.T.C. Wall, Stability, pencils and polytopes, Bull. London Math. Soc. 12 (1980), 401421.
[2] S. Lpez de Medrano, Topology of the intersection of quadrics in R^n, Lecture Notes in Mathematics 1370 (1989), 280292.
[3] S. Gitler and S. López de Medrano, *Intersections of Quadrics, Moment-Angle Manifolds and Connected Sums*, axXiv:0901v2, 2009, Geometry and Topology 2013.

[4] S. López de Medrano, *Singularities of real homogeneous quadratic mappings*, RACSAM, Series A., doi:10.1007/s13398-012-0102-6, 2012.

[5] Y. Barreto, S. López de Medrano and A. Verjovsky, *Moment-angle manifolds, intersection of quadrics and higher dimensional contact manifolds*, arXiv 1305.3208.

[6] V. Gómez Gutiérrez and S. López de Medrano, *Topology of the intersection of quadrics in R^n II*, to appear.