Topological classification and stems of co-rank two map germs from the plane to the plane.

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Abstract

The main purpose of this work is to describe the topological orbits which are in a $\mathcal{K}$-orbit of finitely determined map germs from $\mathbb{C}^2$ to $\mathbb{C}^2$ to. At least for a very large number of co-rank two $\mathcal{K}$-orbits. The co-rank one case is described by Gaffney and Mond in [1, Proposition 4.5. and Theorem 4.6].

The simplest case of $\mathcal{A}$-finitely determined co-rank two map germs is the $\mathcal{K}$-class $\mathcal{K}(xy, x^2 + y^3)$, Gaffney and Mond in [1, Example 5.11] showed that there exists only one topological orbit in this $\mathcal{K}$-class. The next example given by Gaffney-Mond in [1, Example 5.12] is the $\mathcal{K}$-class $\mathcal{K}(xy, x^3 + y^4)$, in this case they expected that the number of topological orbits was finite.

To our surprise, we found a special type of germs in this $\mathcal{K}$-class that are not $\mathcal{A}$-finitely determined, moreover from these germs we showed that there exists a non finite number of $\mathcal{A}$-finitely determined germs in this $\mathcal{K}$-class which belong to different topological orbits. These special germs are called *stems* by D. Mond in [2] and are well known in the class of germs of maps from surfaces to 3-space, the germs S_∞, B_∞ and H_∞.

Therefore, the most natural step in this work was to search for stems in other $\mathcal{K}$-classes. Following this point of view we give a complete answer for this question for any given co-rank two $\mathcal{K}$-class with finitely determined normal form. We show how to obtain stems in any $\mathcal{K}$ class $(xy, x^a + y^b)$, the only exceptions are the cases $(2, 3)$ and $(2, 5)$.

Moreover, from the stems in these $\mathcal{K}$-classes we can show that there exists a non finite number of topological orbits. Again the the only exceptions are the cases $(2, 3)$ with one topological orbit and $(2, 5)$ with two topological orbits.

References

[1] Gaffney T. and Mond D., *Weighted homogeneous maps from the plane to the plane*. Math. Proc. Camb. Phil. Soc., vol. 109, 451–470, 1991.

[2] Mond D. *Some Remarks on the Geometry and classification of germs of maps from surfaces to* 3*-space*. Topology **26**, 3, 1987, 361–383.