On foliations by curves in the projective plane having a very special subscheme of its singularities.

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Abstract

Let $\mathbb{P}^2$ denote the complex projective plane and $\Omega^1_{\mathbb{P}^2}$ and $\Theta_{\mathbb{P}^2}$ its cotangent and tangent sheaves, respectively. A foliation by curves with singularities (a *foliation* in the sequel) of degree r on $\mathbb{P}^2$ is the class

$$\mathcal{F} = [\Omega] \in \mathbb{P}\mathrm{H}^0(\mathbb{P}^2, \Omega^1_{\mathbb{P}^2}(r+2)) \text{ (or } [\alpha] \in \mathbb{P}\mathrm{H}^0(\mathbb{P}^2, \Theta_{\mathbb{P}^2}(r-1)))$$

of a global section $\Omega \in \mathrm{H}^0(\mathbb{P}^2, \Omega^1_{\mathbb{P}^2}(r+2))$. In affine coordinates (X_0, X_1, X_2) of $\mathbb{C}^3$, the section Ω corresponds to a $1-$form $\Omega = \sum_{i=0}^{2} A_i dX_i$, where A_i are homogeneous polynomials of degree $r+1$ satisfying the *Euler condition* $\sum_{i=0}^{2} X_i A_i = 0$.

The *singular scheme* $\mathrm{S}(\mathcal{F})$ of $\mathcal{F}$ is the scheme of zeroes of a section $\Omega \in \mathcal{F}$ and we say that $\mathcal{F}$ has *isolated singularities* if $\mathrm{S}(\mathcal{F})$ is zero-dimensional.

A foliation with isolated singularities is determined by its singular scheme in the following sense: Let $\mathcal{F}$ and $\mathcal{F}'$ be two foliations of degree $r \geq 2$, on $\mathbb{P}^2$. If $\mathcal{F}$ has isolated singularities and $\mathrm{S}(\mathcal{F}') \supseteq \mathrm{S}(\mathcal{F})$, then $\mathcal{F}' = \mathcal{F}$ (see [1]).

Assume that $\mathcal{F}$ has isolated singularities. If moreover $\mathrm{S}(\mathcal{F})$ is reduced, then there exist proper subschemes $Z \subset \mathrm{S}(\mathcal{F})$ which still determine $\mathcal{F}$ in the sense above: if $\mathrm{S}(\mathcal{F}') \supseteq Z$, then $\mathcal{F}' = \mathcal{F}$. Such subschemes were called *special* in [2].

A *very special* subscheme $\hat{Z} \subset \mathrm{S}(\mathcal{F})$ is one that determines $\mathcal{F}$ in the sense above and whose degree $\deg\hat{Z}$ is minimal with respect to this property.

In the talk we will compute the minimal degree $\mu(2, r-1)$ from above and will prove that the set of foliations of degree r whose singular scheme contains a very special subscheme contains a non-empty open set.

Most of this work is join with A. Campillo.

1. A. Campillo, J. Olivares. *On sections with isolated singularities of twisted bundles and applications to foliations by curves*, Math. Res. Lett., 10, 2003, 651-658.

2. A. Campillo, J. Olivares. *Special subschemes of the scheme of singularities of a plane foliation*, C. R. Math. Acad. Sci. Paris, 344, 2007, 9, 581-585.