Meeting in Analysis

Celebrating Salvador Pérez-Esteva's 60th birthday

Program

	November 18	November 19	November 20
9:30–10:20	Alberto Ruiz	Marius Mitrea	Alberto Ruiz
10:30–11:05	Welcome	Carlos Bosch	Steve Hofmann
11:10–11:45	Nikolai Vasilevski	Philippe Jaming	Mónica Clapp
11:50–12:15	Coffee	Coffee	Coffee
12:15–13:05	Marius Mitrea	Alberto Ruiz	Marius Mitrea
13:10–13:45	Martha Guzmán	Michael Shapiro	Santiago López de Medrano
14:00–16:00	Lunch	Lunch	Lunch
16:00–16:35	Hugo Ocampo	Gustavo Ponce	Marcos López
16:40–17:15	James Wright	Oscar Blasco	Wilfredo Urbina
17:20–17:55	Nils Ackermann	Maribel Loaiza	Josefina Álvarez
17:55–18:30			Closure

November 18

Ruiz, Alberto
Universidad Autónoma de Madrid, Spain
Estimates for the resolvent, extensions and applications

Abstract: We will review well known estimates for the resolvent of Laplacian in several spaces. We will extend them to Lamé operator in elasticity, and see some obstructions to the standard extension by using maximal operators and A_p weight. Finally we will see some applications to scattering and unique continuation.

Vasilevski, Nikolai
Departamento de Matemáticas, CINVESTAV, México
Algebras generated by Toeplitz operators on the unit ball

Abstract: We give a detailed description of various algebras, both C* and Banach, that are generated by Toeplitz operators with special kind of symbols acting on the weighted Bergman space over the unit ball.

Mitrea, Marius
University of Missouri, USA
Scattering, Singular Integrals, and Geometric Measure Theory

First lecture: *Scattering, Singular Integrals, and Geometric Measure Theory, I*

Abstract: In this lecture I will answer the following basic question: What are the optimal assumptions, of geometric and analytic nature, which guarantee that a null-solution of the Helmholtz operator in an exterior domain can be represented in terms of layer potentials naturally associated with the said Helmholtz operator and given domain? This work, at the interface between Geometric Measure Theory, Harmonic Analysis, Scattering Theory, and Clifford Analysis, generalizes and unifies classical results of Sommerfeld, Weyl, Müller, and Calderón. It also builds on the work of Salvador Pérez-Esteva and Emilio Marmolejo-Olea published in MMAS, 2013.

Guzmán, Martha
Universidad de Sonora, México
Continuity of Hardy type operators on rectangularly defined spaces

Abstract: We introduce a generalization of rectangular Herz spaces and prove boundedness of a version of the n-dimensional Hardy operator on them. We also study the action of the Hausdorff operator on some subspaces of BMO.

Ocampo, Hugo
Universidad Autónoma del Estado de Morelos, México
Atomic decomposition of functions spaces

Abstract: We made an atomic decomposition for vector Hardy spaces $H^p(\mathbb{R}^n)$, with parameter $0 < p < 1$. The next goal is to obtain projections of spaces conformed by $L^{q,r}$-atoms, spaces localized between the Hardy space $H^1(\mathbb{R})$ and $L^1(\mathbb{R})$, over ortogonal spaces by the use of a multiresolution analysis.

Wright, James
University of Edinburgh, UK
Lebesgue Constants: connections with pointwise ergodic theorems

Abstract: The classical Lebesgue constant for continuous periodic functions is useful in the study of pointwise and uniformly convergent fourier series. We examine variants for functions with a sparse spectrum and in particular we look at extensions to functions of several variables. Interestingly there are some connections with extensions and generalisations of Bourgain's work on pointwise ergodic theorems along sparse subsets of integers.

Ackermann, Nils
Instituto de Matemáticas, Universidad Nacional Autónoma de México, México
Uniform Continuity and Brézis-Lieb Type Splitting for Superposition Operators in Sobolev Space

Abstract: Denote by $\mathscr{F}$ a superposition (or Nemyckii-) operator induced by a continuous function $f\colon \mathbb{R} \to \mathbb{R}$ that satisfies a polynomial growth condition with exponent $\mu > 0$. If $\nu \geq 1$ is such that $\mu\nu > 2$ and that the Sobolev embedding of $H^1(\mathbb{R}^N)$ in $L^{\mu\nu}(\mathbb{R}^N)$ is locally compact we prove that $\mathscr{F}\colon H^1(\mathbb{R}^N) \to L^\nu(\mathbb{R}^N)$ is uniformly continuous on any bounded subset of $H^1(\mathbb{R}^N)$. This result implies a variant of the Brézis-Lieb Lemma that applies to more general nonlinear superposition operators within this range of growth exponents. In particular, no convexity or Hölder continuity assumptions are imposed on f, in contrast to previously known results.

November 19

Mitrea, Marius
University of Missouri, USA
Scattering, Singular Integrals, and Geometric Measure Theory

Second lecture: *Scattering, Singular Integrals, and Geometric Measure Theory, II*

Abstract: In this lecture, I will survey results highlighting the interplay between the geometry of Hardy spaces and functional analytic properties of singular integral operators, such as Riesz transforms and Cauchy-Clifford operators, one the one hand, and on the other hand, the regularity and rigidity of domains of locally finite perimeter. This is related to the work of Salvador Pérez-Esteva et al published in GAFA, 2009.

Bosch Giral, Carlos
Instituto Tecnológico Autónomo de México, México
Salvador, his first steps in mathematics

Abstract: Let E be the inductive limit of an increasing sequence of locally convex space E_n, and suppose that each continuous linear form on E_n extends to a continuous linear form on E_{n+1} and that the closure of E_n in E is contained in E_{n+p}. Then every bounded set of E is contained and bounded in some E_n. This result that generalizes the classical case of strict inductive limits was almost found by Salvador. We will talk about that and Salvador during that time.

Jaming, Philippe
Université de Bordeaux 1, Institut de Mathématiques de Bordeaux UMR 5251, FRANCE
Uniqueness for Discrete Schrödinger Evolutions

Abstract: We prove that if a solution of the discrete time-dependent Schrödinger equation with bounded real potential decays fast at two distinct times then the solution is trivial. For the free Schrödinger operator, as well as for operators with compactly supported time-independent potentials, a sharp analog of the Hardy uncertainty principle is obtained, using an argument based on the theory of entire functions. Logarithmic convexity of weighted norms is employed in the case of general real-valued time-dependent bounded potentials.
Joint work with Yurii Lyubarskii, Eugenia Malinnikova, Karl-Mikael Perfekt (Trondheim)

Ruiz, Alberto
Universidad Autónoma de Madrid, Spain
Estimates for the resolvent, extensions and applications

Abstract: We will review well known estimates for the resolvent of Laplacian in several spaces. We will extend them to Lamé operator in elasticity, and see some obstructions to the standard extension by using maximal operators and A_p weight. Finally we will see some applications to scattering and unique continuation.

Shapiro, Michael
Escuela Superior de Física y Matemáticas, Instituto Politécnico Nacional, México
On a class of holomorphic mappings in $\mathbb{C}^2$ related to bicomplex numbers

Abstract: In classic multidimensional complex analysis, a holomorphic mapping in $\mathbb{C}^2$ is just a pair of holomorphic functions of two complex variables with no relations between its components. It turns out that it is possible to introduce a Cauchy-Riemann-type relation in such a way that the arising subclass of holomorphic mappings possesses a rich theory quite similar to that of functions in one variable. It will be shown that a right way of treating it is via the so-called bicomplex analysis, that is, a study of derivable bicomplex functions. The main peculiarities of this approach will be presented.

Ponce, Gustavo
University of California-Santa Barbara, USA
On special properties of solutions of the k-generalized Korteweg-de Vries equation

Abstract: In 1983 T. Kato established the so called “local smoothing effect" in solutions of the KdV equation. More precisely, he prove that (smooth) solutions of the initial value problem (IVP)

$$\begin{cases} \partial_t u + \partial_x^3 u + u\partial_x u = 0, \quad t,\ x \in \mathbb{R}, \\ u(x,0) = u_0(x) \end{cases} \tag{1}$$

satisfies that for any $T, R > 0$

$$\int_{-T}^{T}\int_{-R}^{R} (\partial_x u(x,t))^2\, dx\, dt \le c(T;R;\|u_0\|_2). \tag{2}$$

Starting with the identity deduced by T. Kato to establish (2) we (joint work with P. Isaza and F. Linares) shall prove :
(i) “sharp" decay estimates for solutions of the IVP (1)
and
(ii) the preservation of some regularity of the data u_0 for the solution flow of (1).
Also we (joint work with F. Linares and D. Smith) shall show that the solution flow does not preserve other kind of regularities exhibited by the initial data u_0.

Blasco, Oscar
Universidad de Valencia, Spain
Averaging operators, Berezin transforms and atomic decomposition on Bergman-Herz spaces

Abstract: We study the class of weight functions W in the unit disk for which the averaging operators $\mathscr{A}_r\phi(z) = \frac{1}{|D(z,r)|}\int_{D(z,r)}\phi(w)dA(w)$ are bounded on $L^p(W)$, where $D(z,r)$ is the disk centered at z and radius r in the hyperbolic metric. We also show the atomic decompositions on weighted Bergman-Herz spaces $A^p_q(W)$ for weights in the above class for which the Bergman projection is continuous on the Herz spaces $\mathscr{K}^p_q(W)$.
(joint work with Salvador Pérez-Esteva)

Loaiza, Maribel
Departamento de Matemáticas, CINVESTAV, México
Commutative algebras of Toeplitz operators on the pluriharmonic Bergman space

Abstract: Toeplitz operators acting on the harmonic Bergman space on the unit disk (or in the upper half-plane) are represented in terms of Toeplitz operators acting on the Bergman space and Toeplitz operators acting on the anti- Bergman space. In spite of this, Toeplitz operators acting on the harmonic Bergman space have some unexpected properties, for example the Fredholm index of every Fredholm Toeplitz operator with continuous symbol is equal to zero, contrary to the case of Toeplitz operators acting on the Bergman space. In this talk we study C^*-algebras generated by Toeplitz operators acting on the harmonic Bergman space of the upper half-plane as well as the generalization of the results to the pluriharmonic Bergman space of the Siegel domain in $\mathbb{C}^n$. In particular we study Toeplitz operators whose symbols are invariant under the action of the nilpotent group of biholomorphisms of the Siegel domain and Toeplitz operators whose symbols are invariant under the action of the of the quasi-parabolic group of biholomorphisms of the Siegel domain.

November 20

Ruiz, Alberto
Universidad Autónoma de Madrid, Spain
Estimates for the resolvent, extensions and applications

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Hofmann, Steve
University of Missouri, USA
Quantitative Rectifiability and boundary behavior of harmonic functions

Abstract: A classical theorem of F. and M. Riesz states that for a simply connected domain in the complex plane with a rectifiable boundary, harmonic measure and arc length measure on the boundary are mutually absolutely continuous. On the other hand, an example of C. Bishop and P. Jones shows that the latter conclusion may fail, in the absence of some sort of connectivity hypothesis.
In this talk, we present a survey of recent developments in an ongoing program to find scale-invariant, higher dimensional versions of the F. and M. Riesz Theorem, as well as converses. In particular, we discuss substitute results that continue to hold in the absence of any connectivity hypothesis. We also discuss related theory for solutions of more general elliptic operators than the Laplacian.

Clapp, Mónica
Instituto de Matemáticas, Universidad Nacional Autónoma de México, México

Entire nodal solutions of a semilinear elliptic equation and their effect on concentration phenomena

Abstract: The aim of this talk is to present some new concentration phenomena for solutions to the problem

$$(\wp_\varepsilon) \qquad \begin{cases} -\varepsilon^2 \Delta u + u = |u|^{p-2} u & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

as $\varepsilon \to 0$, where Ω is a bounded smooth domain in $\mathbb{R}^N$, $N \geq 3$, $\varepsilon > 0$, and $p \in (2, 2^*)$, with $2^* := \frac{2N}{N-2}$ the critical Sobolev exponent.
This problem appears as a model for pattern formation in various branches of science, e.g., in the study of stationary solutions for the Keller-Segal system in chemotaxis or the Gierer-Meinhardt system in biological pattern formation, and it has been extensibly studied.
A common feature of all available results is that the asymptotic profile of the solutions at the blow-up points is a rescaling of the ground states of the limit problem

$$(\wp_\infty) \qquad \begin{cases} -\Delta u + u = |u|^{p-2} u, \\ u \in H^1(\mathbb{R}^N). \end{cases}$$

We show that there are sign changing solutions to $(\wp_\varepsilon)$, which concentrate at a single point, whose asymptotic profile as $\varepsilon \to 0$ is a rescaling of a nonradial sign changing bound state to the limit problem $(\wp_\infty)$.
This is joint work with P.N. Srikanth (Tata Institute of Fundamental Research, Bangalore).

Mitrea, Marius
University of Missouri, USA
Scattering, Singular Integrals, and Geometric Measure Theory

Third lecture: *Scattering, Singular Integrals, and Geometric Measure Theory, III*

Abstract: In this lecture I will present some very recent progress in the direction of developing a Calderón-Zygmund theory for boundary layer potentials associated with higher order systems of differential equations in uniformly rectifiable domains. This is joint work with Gustavo Hoepfner, Paulo Liboni, D. Mitrea, and M. Mitrea.

López de Medrano, Santiago
Instituto de Matemáticas, Universidad Nacional Autónoma de México, México
Topology of the intersection of concentric ellipsoids

Abstract: Intersections of ellipsoids appear in questions of Mechanics of Rigid Bodies, Control Theory (where there are specialized software to compute them) and potentially in Differential Geometry. For more than 30 years I have been studying the topology of the intersections of several concentric ellipsoids (in a different presentation) with some downs and ups. I will relate this development up to the most recent results.

López, Marcos
UCIM, Universidad Nacional Autónoma de México, México
Toeplitz operators with symbols supported in circles

Abstract: In this work we analyzed the dependence on t of the operator-norm and the p-Schatten norms of Toeplitz operators defined on the Bergman space, whose symbols are measures with a positive density supported in the circle of radius $t > 0$.
This is a joint work with Salvador Pérez-Esteva.

Urbina, Wilfredo
Roosevelt University, USA
New real variable methods in H summability of Fourier series

Abstract: In this paper we shall be concerned with H_α summability, for $0 < \alpha \leq 2$ of the Fourier series of arbitrary $L^1([-\pi,\pi])$ functions. The methods to be employed here are a refinement of the real variable methods introduced by Marcinkiewicz. In addition, we introduce maximal theorems with respect to the Lebesgue measure and A_1 weights.
(joint work with Calixto P. Calderón y A. Susana Coré)

Alvarez, Josefina
New Mexico State University, USA
Nonlinear Initial Value Problems With Measure Solutions

Abstract: We discuss two examples of such initial value problems, both involving extensions to measures of Nemickiĭ operators. In the first example, we work with general nonnegative Carathéodory functions and with signed measures that are absolutely continuous with respect to a sigma finite measure. For the second example, our Carathéodory function is piecewise linear, allowing us to extend the associated Nemickiĭ operator to general signed measures. In each case we prove existence and uniqueness of a solution of the initial value problem, using an extension of the Banach fixed point theorem.
This presentation is based on and extends work done in collaboration with María Cristina Mariani (UT El Paso, mcmariani@nmsu.edu), Michael Eydenberg (NSA, mseyden@nmt.edu).