VACUUM ENERGY OF THE ELECTROMAGNETIC FIELD IN A ROTATING SYSTEM

S. HACYAN and A. SARMIENTO
Instituto de Astronomía, Universidad Nacional Autónoma de México, Ap. P. 70-264, México, D.F., Mexico

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The vacuum energy of the electromagnetic field is calculated for a uniformly rotating observer. The spectrum of vacuum fluctuations is composed of the zero-point energy with a modified density of states and a contribution due to the rotation which is not thermal.

It is now generally accepted that the vacuum fluctuations produce some thermal-like effects in an accelerated system. If there is any hope of observing these effects in the future, a uniformly rotating frame seems to be the simplest system which could be studied in a laboratory. In particular, it would be interesting to see whether there are measurable effects associated with the electromagnetic vacuum fluctuations, which are known to produce the Casimir effect and the Lamb shift.

The vacuum stress of a massless scalar field has been analyzed by many authors [1-5]. The aim of the present article is to generalize the results of Letaw and Pfautsch [1] to the case of an electromagnetic field. This is most easily achieved by using the formalism developed in a previous paper [6], which was specially designed for handling the cases of massless fields with arbitrary spin.

The world line of a detector in uniform rotation is described by the parametric equations

$$t=\gamma\tau, \quad x=\gamma R\cos(\Omega\tau), \quad y=\gamma R\sin(\Omega\tau),$$
$$z=0\,, \tag{1}$$

where τ is the proper time, Ω the angular velocity, $v=\Omega R$, $\gamma=(1-v^2)^{-1/2}$, and γR the rotation radius ($c=1=\hbar$ hereafter). Following ref. [6], we calculate the Wightman function $D^{\pm}_{\alpha\beta}(\tau+\frac{1}{2}\sigma, \tau-\frac{1}{2}\sigma)$ at two points on the detector world line (with proper times $\tau\pm\frac{1}{2}\sigma$) and contract it with $u^\alpha(\tau)u^\beta(\tau)$, where $u^\alpha=\mathrm{d}x^\alpha/\mathrm{d}\tau$ is the four-velocity of the detector. The result after some straightforward algebra is

$$u^\alpha(\tau)u^\beta(\tau)D^{\pm}_{\alpha\beta}(\tau+\tfrac{1}{2}\sigma, \tau-\tfrac{1}{2}\sigma)$$
$$=12/\pi(\sigma\mp\mathrm{i}0)^4-2(\gamma v\Omega)^2/\pi(\sigma\mp\mathrm{i}0)^2$$
$$+\pi^{-1}(\tfrac{1}{2}\Omega)^4F_\mathrm{v}(x)\,, \tag{2}$$

where $x=\Omega\sigma/2$ and

$$F_\mathrm{v}(x)=[(1-v^2)/(x^2-v^2\sin^2x)^3]$$
$$\times[(3+v^2)x^2+(1+3v^2)v^2\sin^2x$$
$$-8v^2x\sin x]$$
$$-3/x^4+2\gamma^2v^2/x^2\,. \tag{3}$$

In eq. (2) we have separated the terms containing poles at $\sigma=\pm\mathrm{i}0$, so that $F(x)$ is finite for all real x. The energy spectrum of the field turns out to be

$$\mathrm{d}e/\mathrm{d}\omega=(1/2\pi^2)[\omega^2+(\gamma v\Omega)^2]\omega$$
$$+\frac{\Omega^3}{16\pi^3}\int_0^\infty F_\mathrm{v}(x)\cos[(2\omega/\Omega)x]\,\mathrm{d}x\,. \tag{4}$$

In order to clarify this result, we can compare it with the similar case of a uniformly accelerated observer who detects an energy spectrum given by [7]

$$\mathrm{d}e/\mathrm{d}\omega=\pi^{-2}(\omega^2+a^2)\omega$$
$$\times\{\tfrac{1}{2}+[\exp(2\pi\omega/a)-1]^{-1}\}\,, \tag{5}$$

where a is the acceleration. The zero-point field contributes with a term $(\omega^3+a^2\omega)/2\pi^2$, instead of the usual $\omega^3/2\pi^2$. This is due to a modification of the density-of-states factor. Due to this factor, the energy

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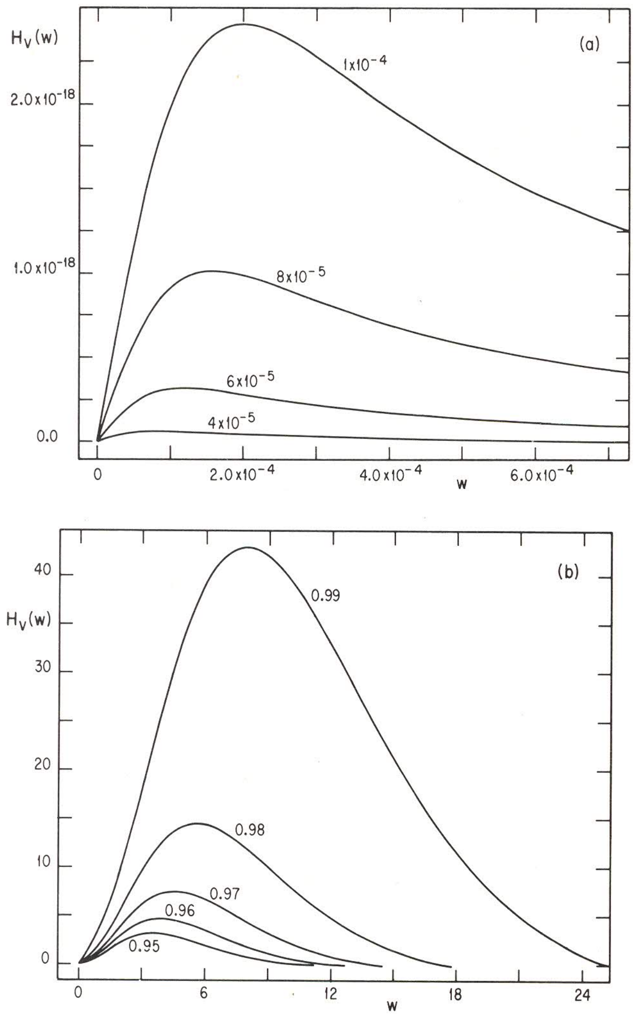

Fig. 1. The spectral function $H_v(w)$ for (a) non-relativistic velocities $v=4\times10^{-5}$–10^{-4}, (b) relativistic velocities $v=0.95$–0.99.

spectrum is not strictly planckian; for instance, $de/d\omega$ has the finite value $a^3/2\pi^3$ if $\omega=0$.

Comparing eqs. (4) and (5), we see that the density-of-states factor has the same form in the two cases, with an acceleration given by $a=\gamma v\Omega$. The integral in eq. (4) is thus the analogue of the thermal function in eq. (5). The integral in eq. (4) has been evaluated numerically and the function defined as

$$H_v(w)=v^3\frac{w^2}{w^2+(2\gamma v)^2}\int_0^\infty F_v(x)\cos(wx)\,dx\,, \qquad (6)$$

(where $w=2\omega/\Omega$) is plotted in fig. 1, for low and high values of v. For the sake of clarity, we have included the factor $\omega^2/[\omega^2+(\gamma v\Omega)^2]$ in order to reproduce functions similar to planckian distributions. Thus, the energy density per unit frequency is

$$\frac{de}{d\omega}=\frac{1}{16\pi^3R^3}\frac{\omega^2+(\gamma v\Omega)^2}{\omega^2}H_v(2\omega/\Omega)\,, \qquad (7)$$

where, as usual, we have discarded the infinite energy associated to the zero-point field. The total energy density of the field can be obtained by integrating eq. (4). The result is

$$e=\frac{\Omega^4}{16\pi^2}F_v(0)=\frac{\Omega^4}{16\pi^2}\gamma^4\,v^2\left(\tfrac{17}{45}+\tfrac{11}{15}v^2\right)\,. \qquad (8)$$

In order to have some feeling of the orders of magnitude involved, we may compare eq. (8) with the corresponding value of the black-body energy density e_{BB}; namely, we evaluate the effective temperature T_{eff} for which $e=e_{BB}$. A simple calculation shows that

$$kT_{eff}=\frac{\hbar}{2\pi R}\left(\frac{v^3}{c}\right)^{1/2}\gamma\left(\frac{17}{3}+11\frac{v^2}{c^2}\right)^{1/4}\,. \qquad (9)$$

This temperature can be significative for a beam of ultrarelativistic electrons in a storage ring (see, e.g., ref. [3]).

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