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Texture and inflation in a closed universe

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We present a cosmological model with a global homogeneous texture and inflation, but without an initial singularity. The Universe starts from an equilibrium configuration in a symmetric vacuum; the dynamic stability of this configuration is studied. We obtain numerical solutions which show that the Universe expands exponentially and the texture field decays in a finite time; this corresponds to a period of inflation followed naturally by a Friedmann expansion.

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Recently Davis has presented an interesting cosmological model in which the Universe can be smaller than what it actually appears to be [1–2]. The basic ingredient is a texture field which is stable due to the curvature of the Universe. On the other hand, the inflationary model of the Universe [3] has been very popular since it solves several problems in cosmology. Can these two models be combined? In order to answer this question at least partially, we present in this Brief Report a cosmological model which combines some of the features of texture and inflation. The result is a hybrid which also has some features of the so-called Eddington-Lemaître model [4]. Our model avoids an initial singularity; instead, the Universe starts from a quasiequilibrium configuration with a finite radius of curvature: its value is of the order of the Planck length. We show that, in such a case, the spatial curvature of the Universe is unstable, but the symmetric vacuum configuration is stable as long as the expansion is slow, the condition for this behavior being exactly the same for the present Universe to be spatially closed. This result does not rely on any assumed *ad hoc* form for the potential of the scalar fields.

The metric of a closed universe is given by

$$ds^2=-dt^2+R^2(t)[d\chi^2+\sin^2\chi(d\vartheta^2+\sin^2\vartheta\, d\varphi^2)] , \quad (1)$$

where χ, ϑ, and φ are the standard angular coordinates.

The Friedmann equations for the scale factor $R(t)$ in a closed universe are

$$\dot{R}^2-(8\pi G/3)R^2(\rho_r+\rho_m+\rho_t)=-1 , \quad (2)$$

where ρ_r, ρ_m, and ρ_t are the radiation, matter, and texture energy densities, respectively. In the early universe, $\rho_r=CR^{-4}$ (C is a constant) and ρ_m is negligible.

Following Davis [1], we consider a texture produced by the breaking of a global symmetry O(4)→O(3) and take as Lagrangian density for the fourplet field ϕ:

$$L_t=\partial^\mu\phi\cdot\partial_\mu\phi+\lambda(\phi\cdot\phi-v^2)^2 . \quad (3)$$

Assuming a *homogeneous* texture, we can choose, as an ansatz,

$$\phi=Q(t)\{\cos\varphi\sin\vartheta\sin\chi,\sin\varphi\sin\vartheta\sin\chi , \cos\vartheta\sin\chi,\cos\chi\} , \quad (4)$$

from which it follows that

$$\rho_t=\frac{1}{2}\left[\dot{Q}^2+\frac{3Q^2}{R^2}\right]+\frac{\lambda}{2}(Q^2-v^2)^2 , \quad (5)$$

and the scalar Q satisfies the equation

$$\ddot{Q}+3\frac{\dot{R}}{R}\dot{Q}+\frac{3}{R^2}Q+2\lambda Q(Q^2-v^2)=0 . \quad (6)$$

In the present universe $Q=v$.

The Friedmann equation with an energy density given by (5) and $Q=v$ was discussed by Davis [1], who shows that the Universe could be closed but mimic an open universe if $\gamma\equiv4\pi Gv^2>1$; this would imply that the symmetry-breaking mass v is of the order of the Planck mass m_P.

Now, Eqs. (2), (5), and (6) can be obtained from the Lagrangian

$$L=-R\dot{R}^2+\frac{4\pi G}{3}R^3\dot{Q}^2-U(R,Q) , \quad (7)$$

where the potential U is given by

$$U(R,Q)=\frac{4\pi G}{3}\left[3RQ^2+\lambda R^3(Q^2-v^2)^2+2\frac{C}{R}\right]-R , \quad (8)$$

with the additional constraint that the conserved energy E (or the Hamiltonian) be identically null: $E=0$.

The Euler-Lagrange equations admit a stationary solution at $Q=0$ and $R\equiv R_0$, where the derivatives of the potential U vanish; physically, this corresponds to a static universe in a symmetric vacuum. Also, the condition $E=0$ fixes the values of $R_0=(3/2\lambda\gamma v^2)^{1/2}$ and

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$C=9/8\lambda\gamma^2$. In order to see whether such a solution is stable, we expand the Lagrangian around the equilibrium solution and obtain the pair of equations:

$$\ddot{R}=\tfrac{4}{3}\lambda\gamma v^2(R-R_0)\ , \tag{9}$$

$$\ddot{Q}=2\lambda(1-\gamma)v^2 Q\ . \tag{10}$$

Thus $R=R_0$ is an unstable solution, but $Q=0$ is stable if $\gamma>1$. The equilibrium position is of the saddle-point type.

In the model we propose here, the closed Universe undergoes an exponential expansion with a time scale

$$\tau=(3/\pi\lambda)^{1/2}m_P/4v^2\ .$$

The solution of Eqs. (2) and (6) for $Q=0$ is

$$R=R_0(1+\varepsilon e^{t/\tau})\ . \tag{11}$$

Choosing a small value for the constant ε, for example, $\varepsilon=10^{-2}$, is equivalent to fixing the expansion of the Universe to be very slow until time $t\approx 0$; during that period, the magnitude of the texture field is stable and oscillates around the value $Q=0$ with a period

$$\tau_Q=[2\lambda(\gamma-1)]^{-1/2}v^{-1}$$

and a certain amplitude A. After $t\approx 0$, the expansion becomes exponential, just as in the usual inflationary scenario, while the field Q still remains for a certain time in the state of symmetric vacuum.

An important feature of solution (11) is the existence of an event horizon with a radius:

$$R(t_1)\int_{t_1}^{\infty}\frac{dt}{R(t)}=\tau(1+\varepsilon e^{t_1/\tau})\ln(1+\varepsilon^{-1}e^{-t_1/\tau}) \tag{12}$$

at time t_1 [5]. In the far past ($t_1\to-\infty$), the size of the horizon is $|t_1|$; as time evolves, the horizon shrinks down to the value τ during the inflation period. Thus, the Universe could well be homogeneous during the slow expansion period ($t<0$), and it makes sense to assume that the texture field was initially homogeneous. The situation is less clear during the inflation period, since the horizon has shrunk to the size of the quantum fluctuations of the metric and of the field, which are both of order τ; in any case, this problem is also present in the standard inflationary model.

Typical solutions are presented in Fig. 1 for $\lambda=1$ and three values of v/m_P: $\frac{1}{3}$, $\frac{2}{3}$, and 1. At $t=0$, we have taken the initial values $R/R_0=1.01$, $Q=0$, and three different values for the amplitude: $A=10^{-3}$, 10^{-4}, and 10^{-5}. The general behavior is the following.

After $t=0$, the scale factor expands very rapidly with a time scale of order τ while the field Q still makes a few oscillations around the value 0. Then, after this period of inflation, the texture field falls to the value $Q=v$, and undergoes very strong oscillations (which would correspond to a large number of Higgs particles). Such a behavior is similar to the one found by Hawking and Moss [6] for inflation driven by a scalar field.

An important point we want to stress is that the total expansion during the inflationary era is extremely sensitive to the time at which Q reaches the value v; this time, in turn, depends on the value of v and on the amplitude A: the bigger v or the smaller A, the longer Q stays close to zero. We have found, for instance, that in the case $v=m_P$ and $\lambda=1$, the scale factor reaches a value which can be fitted by the formula

$$R\sim R_0(m_P/A)^{6.6}\times 10^6\ .$$

The Universe will presumably supercool during inflation and then reheat producing a large amount of entropy [3,6]. The subsequent evolution until the present time is just as in the case of an open universe, as described by Davis [1]. An interesting property of the model is that the transition from an inflationary to a Fried-

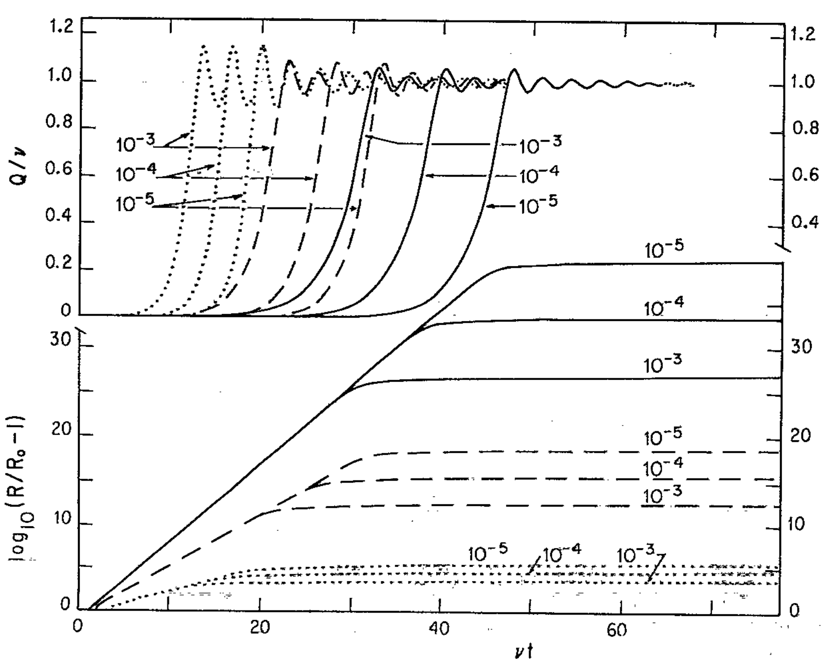

FIG. 1. The upper panel shows the evolution of the field Q with time for the values $\lambda=1$ and $(v/m_P)=\frac{1}{3}$ (dotted line), $\frac{2}{3}$ (broken line), and 1 (solid line). The corresponding values of the oscillation amplitude A are also shown. The lower panel shows the corresponding evolution of the scale factor R with time for the same values of λ, v/m_P, and A. R/R_0-1 is shown in the figure in order to display the exponential expansion near $t\sim 0$ in accordance with Eq. (11); base 10 logarithms are used.

mann universe takes place naturally, without relying in any particular form on the potential or a fine-tuning of v; it is only necessary that the oscillation amplitude of the texture field be small enough during the preinflationary period in order for the field to stay long enough in the state of symmetric vacuum.

The aim of this Brief Report is to show that it is possible to construct a model of a closed Universe with the essential features of inflation and texture. One of the basic ingredients is a gauge field which can be mapped into a spatial S^3 manifold, and texture is such a field. Probably the weakest point of our model is the ansatz used for the texture field, since it implies that the symmetry breaking and the field oscillations take place homogeneously. The homogeneity of the texture field was also assumed by Davis in his original model [1]; the possibility of a nonhomogeneous texture, however, seems to be more interesting [7] (but see Refs. [8,9]). In a more realistic scenario, one would expect symmetry breaking to take place randomly and form topological defects. This is the essential point to be taken into account in any improvement of the present model.

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