Deterministic transport in ratchets

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(Received 27 August 1998; revised manuscript received 14 December 1998)

We present the deterministic transport properties of driven overdamped particles in a simple piecewise-linear ratchet potential. We consider the effects on the stationary current due to local spatial asymmetry, time asymmetry in the driving force, and we include the possibility of a global spatial asymmetry. We present an extremely simple scheme for evaluating the current that is established on the ratchet within an ''adiabatic'' approximation, and compare the results with exact numerical integration of the process.
[S1063-651X(99)01305-7]

PACS number(s): 05.40.−a, 05.60.−k

I. INTRODUCTION

Many interesting aspects of transport properties of particles in ''thermal ratchets'' have been recently reported [1–10]. In their simplest versions, these models consist of independent overdamped particles in a piecewise-linear ''sawtooth'' potential and subject to an oscillating driving force and thermal noise (both with zero mean); these models have received the name of ''periodically rocked ratchet'' in the literature [3,7]. The crucial point in these models is that they are not symmetric under the exchange $x \to -x$. The superimposition of the broken symmetry and the driving force gives rise to a stationary flux even in situations where pure thermal noise would not produce one. As the symmetry can be broken in the potential, in the driving force, or in both, it is possible to obtain a nonmonotonic dependence of the flux on the driving strength and even one or more flux reversals.

A usual approach to the periodically rocked ratchet problem is to write the Fokker-Planck equation for the process [14], and to try to determine from it the flux properties within certain approximations. This approach is quite successful in describing the system consisting of a piecewise-linear potential and a piecewise constant driving force [1], in the approximation of very low driving frequency. As it turns out, most of the interesting behavior of these thermal ratchets is due to the deterministic mechanical transport properties of the ratchet, the effects of the thermal bath being important only when the deterministic current is extremely small or zero. This is in contrast with other ratchet models, for example one where the asymmetric periodic potential is turned on and off periodically, the so-called ''flashing ratchet'' model [7,11–13], where the overdamped deterministic motion never induces transport.

The main point of this work is to study the deterministic behavior of overdamped transport in periodically rocked ratchets. A comparison of our results with results for ''thermal'' ratchets may serve as an aid to establish to what extent the ''thermal bath'' affects and/or determines the interesting transport properties of these systems.

Specifically, we study the deterministic transport properties of a general sawtooth ratchet under an asymmetric driving force that oscillates with a very low frequency. We propose a very simple scheme for calculating the current, and compare our results to the direct numerical integration of the motion. As expected, most of the features found in thermal ratchets are already present in the deterministic case. As our treatment is so straightforward, we can easily generalize it to include (i) random duration asymmetric driving forces (i.e., not necessarily periodic in time) and (ii) the case in which the spatial symmetry is broken also at a global scale (i.e., there is a global drift). We find that it is possible to have transport against the drift, and up to two reversals of the current.

II. THERMAL VS DETERMINISTIC BEHAVIOR

By transport in a thermal ratchet, the overdamped motion of a test particle in a periodic potential $V(x)$ with broken spatial symmetry and subject to a driving force $F(t)$ in the presence of a Gaussian noise $\xi(t)$ obeying $\langle \xi(t)\xi(s)\rangle = 2kT\delta(t-s)$ is understood. The corresponding Langevin equation is then

$$\dot{x} = -\frac{\partial V(x)}{\partial x} + F(t) + \xi(t). \tag{2.1}$$

A deterministic ratchet is defined identically but in the absence of the thermal noise.

Let us now consider the thermal ratchet model by Magnasco [1] as extended to incorporate periodic driving forces which possess temporal asymmetry by Chialvo and Millonas [2]. For the piecewise-linear potential they considered, the average current is

$$\langle J\rangle = \frac{1}{\tau}\int_0^\tau J(F(t))dt$$
$$= \frac{1}{2}\left[(1+\epsilon)J(A) + (1-\epsilon)J\left(-\frac{1+\epsilon}{1-\epsilon}A\right)\right] \tag{2.2}$$

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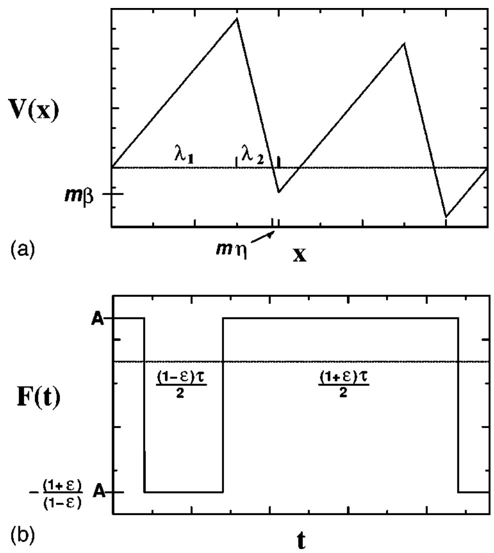

FIG. 1. (a) Piecewise-linear potential $V(x)$ as a function of the position x; its period is $\lambda=\lambda_1+\lambda_2\neq 1$. The two possible spatial asymmetries are sketched in terms of the parameters η for the local asymmetry and β for the global asymmetry (both defined in the text). (b) Periodic external driving force $F(t)$ with a temporal asymmetry but zero mean value.

for a zero mean driving force, $\langle F(t)\rangle=0$, given by

$$F(t)=\begin{cases} -\dfrac{1+\epsilon}{1-\epsilon}A & \text{if} \quad 0\leq t<\dfrac{1}{2}\tau(1-\epsilon), \\ A & \text{if} \quad \dfrac{1}{2}\tau(1-\epsilon)<t\leq\tau, \end{cases} \tag{2.3}$$

where $-1<\epsilon<1$, and $\epsilon=0$ implies a symmetrical driving force [Fig. 1(b)].

Let us recall that for the approximations in this approach to be valid, the driving force $F(t)$ has to be applied during a long lapse in either direction, i.e., the driving frequency has to be very low; for the case described by Eq. (3), this means that τ must be larger than any diffusive relaxation time in the model. As we shall see, this approximation entails another crucial assumption: the distances traveled by a particle in each direction are much larger than the size of the period of the potential.

In order to analyze the role of the thermal bath, we take the limit of the average current as the temperature goes to zero, i.e.,

$$\langle J\rangle_0=\lim_{kT\to 0}\langle J\rangle = \frac{1}{2}\left[(1+\epsilon)\lim_{kT\to 0}J(A)+(1-\epsilon)\lim_{kT\to 0}J\left(-\frac{1+\epsilon}{1-\epsilon}A\right)\right]. \tag{2.4}$$

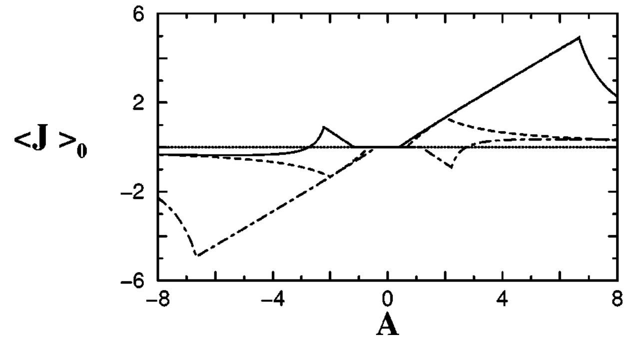

FIG. 2. Limit values of the average current $\langle J\rangle_o$ when the temperature vanishes as a function of the amplitude A of the temporally asymmetric driving force for a fixed value of its degree of asymmetry $\epsilon=1/2$. The values of $\delta=\lambda_1-\lambda_2$ are $-7/10$ (solid line), 0 (dashed line), and 7/10 (dot-dashed line). Note the symmetry $\langle J(-A,-\epsilon)\rangle_o=-\langle J(A,\epsilon)\rangle_o$.

Now, it turns out that

$$\lim_{kT\to 0}J(F) = \begin{cases} 0 & \text{if} \quad F_1\leq F\leq F_2 \\ \dfrac{1}{\lambda}\left[F-\dfrac{Q^2}{\delta Q+\lambda_1\lambda_2 F}\right] & \text{if} \quad F<F_1 \text{ or } F>F_2, \end{cases} \tag{2.5}$$

where

$$F_{\substack{1\\2}}=\frac{Q}{2\lambda_1\lambda_2}[\mp(\delta^2+4\lambda_1\lambda_2)^{1/2}-\delta], \tag{2.6}$$

so that $\langle J\rangle_0$ has the behavior shown in Fig. 2 for different values of δ and $\epsilon=1/2$.

From this graph it is immediately seen that the main features of the average current, its nonmonotonic behavior, its asymptotics, and the possibility of its reversal, are present in the dynamics of the ratchet when the temperature is null. These observations give two suggestions. (i) The behavior of a deterministic ratchet (defined identically but in the absence of thermal noise) could well explain most of the transport properties when it is immersed in a thermal bath, and (ii) the thermal bath will play a definitive role only when the deterministic average current is vanishing.

The features of the average current when the temperature vanishes can be easily derived in terms of a very simple scheme that we present in the following. Our results show that *the qualitative behavior of a thermal ratchet is indeed dictated by the dynamics of the corresponding deterministic ratchet, from which we conclude that the thermal bath modifies the rachet's behavior when the average current is zero*, i.e., for the case of the previous model ratchet, when the force amplitude is in the interval $[F1,F2]$, where $F1$ and $F2$ are the roots defined by Eq. (2.6).

III. TRANSPORT IN A DETERMINISTIC RATCHET

For our analysis of the deterministic transport in ratchets, we have developed a very simple approach and a simple computer program for the calculation of the net current. To begin with, let us take a system composed of a very large number of teeth, all identical in shape and orientation (Fig. 1). There are two kinds of spatial asymmetry in this ratchet: (i) a local asymmetry that we shall specify by a parameter denoted as η, and (ii) a global asymmetry that induces an overall slope to the potential. The parameter that describes the local spatial asymmetry is defined by

$$\eta=\frac{m_2-m_1}{m}, \tag{3.1}$$

while the parameter that characterizes the global asymmetry is

$$\beta=\frac{m_2\lambda_2-m_1\lambda_1}{m\lambda}, \tag{3.2}$$

where $m=m_1+m_2$ and $\lambda=\lambda_1+\lambda_2$. Note that (i) the spatial periodicity of the potential, λ, is no longer taken to be equal to unity, and (ii) the parameter previously used for the description of the local spatial asymmetry, δ, is related to the new one η through the value of the parameter that specifies the global spatial asymmetry, β.

We also consider the effect of a temporal asymmetry in the driving force $F(t)$, which modifies the behavior of the ratchet. Equation (2.3) and Fig. 1 correspond to the case of a periodic driving force, for which the asymmetry is specified by the parameter ϵ, which determines the fraction of the time the force acts in each direction, the so-called periodic, rocked ratchet [3,7]. A different scenario for which this approach is also appropiate is the case in which the driving forces are not periodic but rather are applied in alternating directions for intervals of time with random duration. To ''build in'' asymmetry in this case, we choose the intervals in which the force is applied to the right from a distribution $P_+(\tau)$ (with mean $\langle\tau\rangle_+$), and those in which it is applied to the left from $P_-(\tau)$ (with mean $\langle\tau\rangle_-$). To insure that the time average of the force is zero, the amplitude of the forces applied to the right is $(\langle\tau\rangle_-/\langle\tau\rangle_+)A$, where A is the amplitude of the forces applied to the left. The asymmetry parameter in this case is given by $\epsilon=(\langle\tau\rangle_- - \langle\tau\rangle_+)/(\langle\tau\rangle_- + \langle\tau\rangle_+)$. Both cases, periodic and random duration driving force, are indistinguishable in the following discussion.

For this simple ratchet, motion of a test particle will take place when the amplitude $(1+\epsilon)A/(1-\epsilon)$ of the piecewise-linear driving force applied to the right, say, during a lapse θ_+, is enough to overcome the tooth height. If such is the case, the speed of the test particle during the upward part of the trip will be $v_1=(1+\epsilon)A/(1-\epsilon)-m_1$, where m_1 is the absolute value of the first (ascending) slope, and $v_2=(1+\epsilon)A/(1-\epsilon)+m_2$ during the downward trip, where m_2 is the absolute value of the second (descending) slope. The total time required to travel over one tooth is then $t_+=\lambda_1/v_1+\lambda_2/v_2$. After the time θ_+ has elapsed, a force in the opposite direction is applied during a time θ_-. If the amplitude A is not large enough to overcome the second slope of the tooth (i.e., $A\leqslant m_2$), then the particle will not move until θ_- has elapsed and the force to the right is applied again. If A is larger than m_2, then motion to the left occurs and the time needed to traverse a tooth in the leftward direction will be $t_-=\lambda_2/v_3+\lambda_1/v_4$, where $v_3=A-m_2$ and $v_4=A+m_1$.

Thus, if the force applied to the right is large enough to induce motion, the number of teeth the particle can traverse during the time θ_+ for which it is applied is $N_+=\theta_+/t_+$ and otherwise $N_+=0$. Similarly, if the force to the left induces motion, then it traverses $N_-=\theta_-/t_-$ teeth during the time it is applied. The distances traveled in each direction will then be given by $(\lambda_1+\lambda_2)N_+$ and $(\lambda_1+\lambda_2)N_-$, respectively. Clearly these are approximate values, as the turning points of the motion when the force changes sign, do not coincide with the bottom of a tooth. Nevertheless, the error in the total length traveled will be, at most, of the order of the size of a tooth. Thus the results obtained will be increasingly accurate as the intervals $\theta_\pm$ go to infinity, i.e., the driving frequency goes to zero. It thus becomes apparent that neglecting the contributions of incompletely traversed teeth is an important component of the ''adiabatic'' approximation.

The net current to the right (left), S_- (S_+), is the sum of the traveled distances divided by the sum of the corresponding elapsed times, and the total average current can then be obtained from

$$\langle S\rangle=\frac{(1-\epsilon)S_- - (1+\epsilon)S_+}{2}. \tag{3.3}$$

The explicit expression for the current in this general model is

$$\langle S\rangle=\frac{m}{8}\begin{cases} 0 & \text{if}\quad a<\dfrac{1-\epsilon}{1+\epsilon}\dfrac{m_1}{m}\ \text{and}\ a<\dfrac{m_2}{m},\\[2ex] -(1+\epsilon)\dfrac{4a^2-\eta(4a-\eta)-1}{a+\beta-\eta} & \text{if}\quad \dfrac{m_2}{m}\leqslant a<\dfrac{1-\epsilon}{1+\epsilon}\dfrac{m_1}{m},\\[2ex] \dfrac{4(1+\epsilon)^2a^2+\eta[4(1-\epsilon^2)a+(1-\epsilon)^2\eta]-(1-\epsilon)^2}{(1+\epsilon)a-(1-\epsilon)(\beta-\eta)} & \text{if}\quad \dfrac{1-\epsilon}{1+\epsilon}\dfrac{m_1}{m}\leqslant a<\dfrac{m_2}{m},\\[2ex] \dfrac{8(1+\epsilon)\beta a^2-2(\eta^2-1)[2\epsilon a-(1-\epsilon)(\beta-\eta)]}{(1+\epsilon)a^2+(\beta-\eta)[2\epsilon a-(1-\epsilon)(\beta-\eta)]} & \text{if}\quad \dfrac{1-\epsilon}{1+\epsilon}\dfrac{m_1}{m}\leqslant a\ \text{and}\ \dfrac{m_2}{m}\leqslant a, \end{cases} \tag{3.4}$$

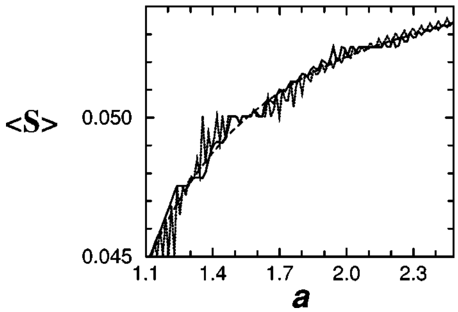

FIG. 3. An example of the differences between the actual numerical integration and the smooth dashed line given by formulas (3.4). The dotted and the uneven solid lines correspond to two numerical calculations carried out with a certain value for τ and half this value, correspondingly.

in terms of the normalized amplitude of the driving force, $a=A/m$.

From this expression, and under the appropriate choice of parameters, one can reproduce the limiting behavior of the average current when the temperature vanishes in the model ratchet described in Sec. II [$\langle J\rangle_o$, Eqs. (2.4), (2.5), and Fig. 2]. The appropriate parameters are $m_1=1/\lambda_1$, $m_2=1/\lambda_2$, $\lambda=1$, and $\beta=0$.

The dimensionless parameters mentioned above, η, β, and ϵ, have been chosen because of their clear physical interpretation, and because the vanishing of their values allows a full recovery of the symmetrical ratchet. Each one of the three symmetry-breaking possibilities (two spatial and one temporal) will be explored in the following.

Since the above expressions are approximate, a simple exact numerical simulation of the process was performed in which we confirmed that indeed the exact current approaches the predictions above as τ increases (see Fig. 3).

Previous work that explicitly studied some aspects of the deterministic behavior of periodically rocked ratchets has been reported in [3,10,12], although without considering the global spatial asymmetry. In both the first and the last of these studies, the driving force was a sinusoidal function of time, and although this implies that their results cannot be directly compared with the outcome of this work, the limit at very low driving frequencies of their results agrees qualitatively with the behavior described in what follows (cf. Figs. 3 of [3], and Figs. 3 and 6 of [12]). The other study [10] analyzes a driving force similar to the one considered in this work [Eq. (2.3)], and the main conclusion reached is that the presence of a thermal bath plays an important role when there is no current or it is very small, since then the thermal fluctuations may benefit from the (temporal or spatial) asymmetries to produce a current or to modify its sense.

A. The local spatial asymmetry

Keeping β and ϵ equal to zero, we have that a positive value of η means a tooth oriented to the right, i.e., one that favors motion to the right, and that negative values of η mean a current to the left. The effect of the local spatial asymmetry is presented in Fig. 4, where the average current $\langle S\rangle$ is plotted as a function of the normalized amplitude of the driving force a for different values of η and keeping both β and ϵ equal to zero. The upper half of the graph shows the effect of varying λ_2 and m_2 (while keeping λ_1 and m_1 constant), and the lower half shows the effect of varying λ_1 and m_1 (for fixed values of λ_2 and m_2).

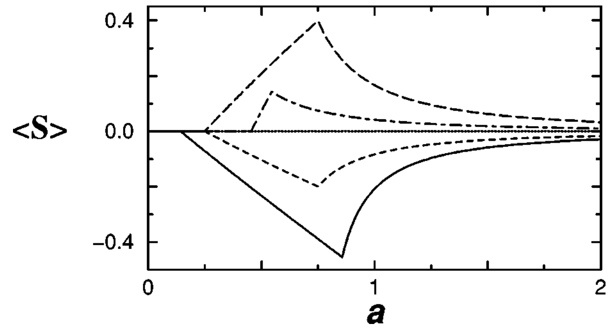

FIG. 4. Effect of the local spatial asymmetry on the motion in the ratchet. The average current $\langle S\rangle$ is shown as a function of the normalized amplitude of the driving force a for different values of η; both β and ϵ are kept equal to zero. The corresponding values of η are 1/2 for the long-dashed line, 1/11 for the dot-dashed line, $-1/2$ for the dashed line, and $-5/7$ for the solid line. Only positive values of a are ploted since $\langle S(-a)\rangle=\langle S(a)\rangle$ in this case.

The maxima of the current occur at a particular value of the normalized amplitude a; this value is given by m_2/m (m_1/m) for positive (negative) values of η. Interchanging the role of the λ's and of the m's, which amounts to reversing the orientation of the ratchet, produces a current which is exactly the same but with the opposite sign (as if reflected by the horizontal axis).

B. The global spatial asymmetry

First, let us note that when β is positive, there is a tendency to ''drift'' to the right caused by the ''global slope'': $(m_2\lambda_2-m_1\lambda_1)/\lambda$, and when β is negative, the ''drift'' is to the left. This underlies the fact that the current does not vanish when the amplitude of the driving force increases unboundedly, but tends to a finite value given by the ''global slope.'' This effect has two consequences: (i) it makes it possible to eventually reverse a current that runs in a certain direction for a wide range of a values; and (ii) it also allows

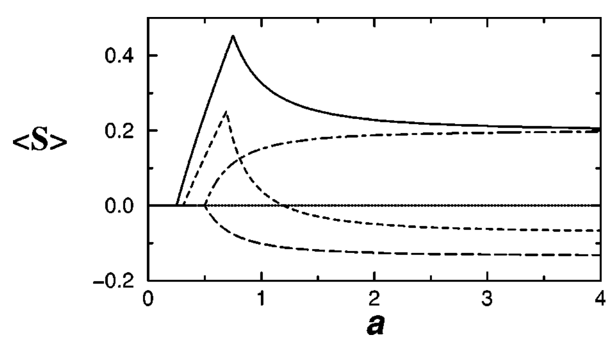

FIG. 5. The effect of a global spatial asymmetry $(\beta\neq 0)$; note that a necessary condition for a current reversal to appear is an η value $\neq 0$. The corresponding values for (η,β) are (1/2,3/20) for the solid line, (0,1/10) for the dot-dashed line, $(5/13,-72/1079)$ for the dashed line, and $(0,-1/10)$ for the long-dashed line; the temporal symmetry remains unbroken. As in the case shown in Fig. 4, $\langle S(-a)\rangle=\langle S(a)\rangle$.

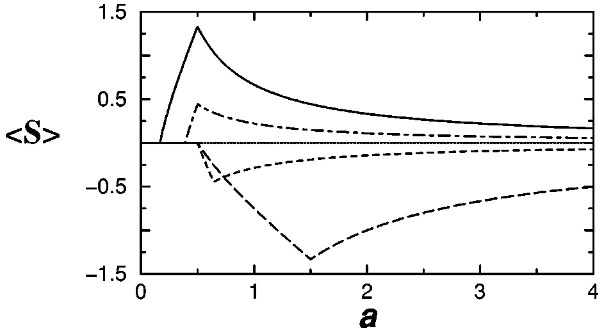

FIG. 6. Effect of the local temporal asymmetry on the average current $\langle S\rangle$ as a function of the normalized amplitude of the driving force a for different values of ϵ; both β and η are kept equal to zero. Only positive values of a are plotted since $\langle S(-a)\rangle = -\langle S(a)\rangle$ [recall that when $\beta=0$, but $\eta\neq 0$, $\langle S(-a,-\epsilon)\rangle = -\langle S(a,\epsilon)\rangle$]. Lines shown correspond to ϵ values of 1/2 (solid line), 1/8 (dot-dashed line), $-1/8$ (dashed line), and $-1/2$ (long-dashed line).

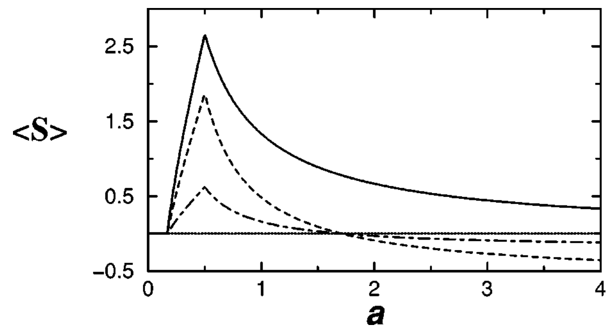

FIG. 8. Combined effect of the temporal asymmetry and the global spatial asymmetry on the average current $\langle S\rangle$ as a function of the normalized amplitude of the driving force a; the value of η is kept equal to zero, ϵ is kept equal to 1/2, and the values of β are 0 (solid line), $-1/10$ (dashed line), and $-1/10$ (dot-dashed line). Note that a β value different from zero is necessary and sufficient for a current reversal to occur, and the presence of the symmetry $\langle S(-a,-\beta)\rangle = -\langle S(a,\beta)\rangle$ in this case.

for a second inversion of the current when both the local spatial and the temporal symmetries have been broken (see Fig. 9).

Examples of the behavior of the current for some values of η and β are presented in Fig. 5, where it is clear that for a flux reversal to occur, it is necesary to have values of η different from zero.

C. The temporal asymmetry

The role of a zero mean, temporally asymmetric driving force in various guises has been previously studied and analyzed; it may increase the current due to a local spatial asymmetry, it may counterbalance it, or it may reverse its flow, depending on the value of ϵ and, of course, on the degree of spatial asymmetry. Figure 6 shows the average current $\langle S\rangle$ as a function of the normalized amplitude a for different values of ϵ; neither of the spatial symmetries is broken.

The average current $\langle S\rangle$ attains a maximum value when $a=m_2/m$ and it must be noted that it does vanish when the amplitude of the driving force tends to infinity, although it does so much more slowly than in the case of a local spatial asymmetry alone.

The following figures show the combined effect on the average current caused by the temporal and local spatial asymmetries (Fig. 7), and by the temporal and global spatial asymmetries (Fig. 8). From Fig. 8 one can confirm that when $\eta=0$, the maximum value of $\langle S\rangle$, and the value of $\lim_{a\to\pm\infty}\langle S\rangle$ are monotonic functions of the global slope, the value of a where the current reversal occurs does not change; this last value is determined by ϵ. Finally, Fig. 9 shows some examples of the behavior shown by the current when the three symmetries have been broken. As already mentioned, it is interesting to note that there may be a second flux reversal in this case.

IV. DISCUSSION

A nonexhaustive analysis of overdamped transport, within an ''adiabatic'' approximation in a simple ratchet, has been presented together with a discussion of the different effects caused by the lack of either spatial or temporal symmetries and the character of the driving force (periodic or stochastic). The features of the flux that we obtain in this work (the nonmonotonicity as a function of the parameters, the sign reversals, and the asymptotic behavior) underly most of the properties of thermal ratchets. Thus, the role of the thermal bath will be important mostly in those regions where the deterministic current is close to or vanishing, otherwise it

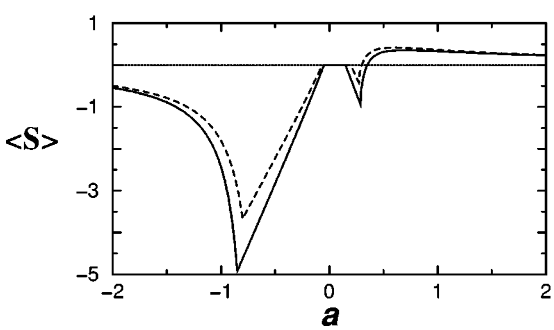

FIG. 7. Combined effect of the temporal asymmetry and the local spatial asymmetry on the average current $\langle S\rangle$ as a function of the normalized amplitude of the driving force a; the value of β is kept equal to zero, and the (η,ϵ) values are $(-7/10,1/2)$ for the solid line and $(-3/5,1/2)$ for the dashed line.

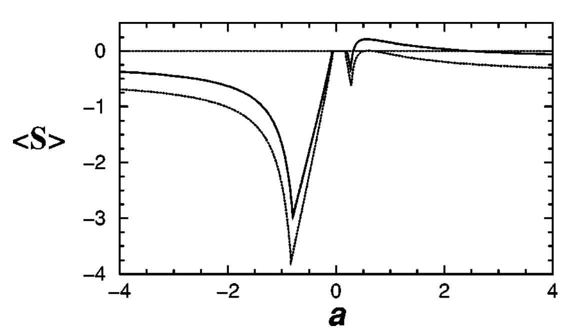

FIG. 9. A double flux reversal may occur in the average current when the three symmetries are broken, and their corresponding parameters have adequate values; $\epsilon=1/2$ and the values of (η,β) are $(-3/5,-4/105)$ for the solid line, and $(-2/3,-19/246)$ for the dotted line.

represents a small perturbation to the deterministic behavior; its effect will be to smooth the sharp edges and to flatten the shape of the curve for the current (cf. Figs. 2 and 3 of Chialvo and Millonas [2]). A possible exception is the case in which the deterministic transport generates a flux against the global bias while a thermal noise, if present, would induce a flux in the direction of the global bias; a noise induced flux reversal could be expected in this situation. Further work considering driving forces with finite frequency, i.e., not within the adiabatic approximation, will be presented in a separate study [15].

Also, work on disordered ratchets is currently under way. In these systems, the role of noise could be essential because deterministic transport may not give rise to a stationary flux, even if the disordered ratchet is statistically asymmetric. The possibility of deterministic transport will obviously depend on the nature of the disorder. If, for example, disorder is introduced by taking the heights of the ratchet teeth from a bounded distribution, then sufficiently strong driving will indeed give rise to deterministic transport. On the other hand, if disorder is introduced by placing p teeth with one orientation and $1-p$ with the opposite orientation, thus statistically breaking the x to $-x$ symmetry, then no periodic driving can generate a flux. The reason for this is that for any periodic driving there will exist, with probability greater than zero, configurations of teeth within which a deterministic periodic ''orbit'' occurs. These orbits work as barriers to deterministic motion, and as they occur with finite density, a flux cannot be sustained in these systems. Thermal noise will play a crucial role in this situation, as it will allow particles to ''overcome'' the barriers and reestablish a flux.

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