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Fractal Dimension in Butterflies' Wings:

a novel approach to understanding wing patterns ?

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Abstract. The geometrical complexity in the wings of several, taxonomically different butterflies, is analyzed in terms of their fractal dimension. Preliminary results provide some evidence on important questions about the (dis)similarity of the wing patterns in terms of their fractal dimension. The analysis is restricted to two groups which are widely used in the literature as typical examples of mimicry, and a small number of unrelated species, thus implying the consideration of only a fraction of the wing pattern diversity. The members of the first mimicry

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ring, composed by the species *Danaus plexippus* (better known as the monarch butterfly), and the two subspecies *Basilarchia archippus obsoleta* (or northern viceroy) and *Basilarchia archippus hoffmanni* (or tropical viceroy), are found to have a very similar value for the fractal dimension of their wing patterns, even though they do not look very similar at first sight. It is also found that the female of another species (*Neophasia terlootii*), which looks similar to the members of the previous group, does not share the same feature, while the *Lycorea ilione albescens* does share it. For the members of the second group of mimicry related butterflies, the *Greta nero nero* and the *Hypoleria cassotis*, it is shown that they also have very close values for the fractal dimension of their wing patterns. Finally, it is shown that other species, which apparently have very similar wing patterns, do not have the same fractal dimension. A possible, not completely tested hypothesis is then conjectured: the formation of groups by individuals whose wing patterns have an almost equal fractal dimension may be due to the fact that they do share the same developmental raw material, and that this common feature is posteriorly modified by natural selection, possibly through predation.

1. Introduction

Motivated by a debate over the possible existence of a mimicry ring in the *Danaus* genus and the subjective character of the term 'similar', an analysis of the geometrical pattern in their wings was carried out using a concept from modern mathematics, the well-known measurement of the fractal dimension of an object via the box-counting method. Examples of co-evolutive paths are now common in text books [4,8,17], and part of the debate can be found in the work of Ritland and Brower [27,28], where they claim that northern viceroys (*obsoleta*) are as unpalatable to their predators as monarchs, at least for butterfly populations found in the state of Florida (United States of America) and therefore, the mimicry should not be considered as a batesian one (where a group serves as a model for another group that imitates a precise, beneficial feature) but rather a müllerian one (where both groups influence one another and eventually reach a common, advantageous characteristic). There is no field test that experimentally could decide the debate in a similar way to the one carried out by Kapan [16] for the *Heliconius* butterflies.

The butterflies in the *Danaus* ring co-inhabit the central regions of México but only the monarch butterflies (*Danaus plexippus*) migrate immense distances ($4,000$ to $5,000$ kms approx.) to the north on the east half of the continent. The tropical viceroy subspecies (*Basilarchia archippus hoffmanni*) was first cataloged by R. L. Chermock in 1947 and is only found in the Mexican regions around the tropic of cancer, while the northern viceroy subspecies (*Basilarchia archippus obsoleta*) was originally cataloged by W. H. Edwards in 1882 and inhabits the eastern regions of North America. The *Basilarchia archippus* butterflies were posteriorly classified as *Limenitis archippus*, unnecessarily duplicating its registry (information on who and when cataloged a (sub)species, is given only for completion but no reference is listed in the corresponding section).

In what follows, we shall show that the viceroy butterflies, and the *Lycore ilione albescens* (classified by Distant in 1876), do share with the monarch butterflies a remarkable feature of the geometrical pattern, its fractal dimension (possibly as a defensive tactic); the females of the *Neophasia terlootii* (classified by Behr in 1869), while looking very similar at first sight, do not have a close fractal dimension. The particular character of the mimicry, if any, is a question we can not answer with the tools used in our analysis. As explained in the next section, the fractal dimension contains geometrical information from all possible scales, and thus, even if the appearance is not so striking at first sight, a common value of the fractal dimension implies an intrinsically higher relationship than mere similarity, possibly an indication of a phylogenetic relationship. This last possibility emerges from the fact that we shall also show that the measurement of the fractal dimension of the geometrical patterns in the wings of butterflies may be an adequate tool for classifying them and for searching candidates to forming groups where the hypothesis on the existence of a common developmental raw material could be fully tested and where the modifications due to subsequent natural selection could be assessed.

2. Fractal dimension

Many natural objects show partial self-similarity in their structure, a feature indicated by a repetition of a pattern at different scales and where a part or several parts of the object resemble the whole. These objects were named fractals by Mandelbrot [22], and the inter-relation between the number of repetition and reduction of an individual, fractal object, constitutes its fractal dimension [30]. Fractal dimension is thus related to the ability for using (filling, in some cases) Euclidean space in an optimal way [14], and this fact is used by several methods for numerically calculating the fractal dimension (box-counting method, mass method). The fractal concept has been widely used in biological sciences, from the human discrimination of fractal images [18] to the origin of allometric scaling laws [29], the varying body size scaling relationships for mammal and bird home ranges [13], and the interdependence of the human portal vein and the hepatic vein in 3D [12]. To the best of our knowledge, the closest to the following analysis is the work by Boyajian and Lutz [3] on the evolution of biological complexity and its relation to taxonomic longevity in the Ammonoidea (see also [7]).

Now, according to the well known box-counting method, the fractal dimension (denoted by D) of a black object over a white background (or *vice versa*), immersed in a two-dimensional environment, is defined as [1,23]:

$$D = \lim_{\varepsilon \to 0} \frac{\ln N(\varepsilon)}{\ln 1/\varepsilon}, \tag{1}$$

where $N(\varepsilon)$ is the number of boxes in a square grid of side-size ε required to cover the object in question. This definition comes from the scaling law $N(\varepsilon) = C(1/\varepsilon)^D$, in which our knowledge on integer-dimension objects is clearly expressed (one needs c/ε boxes of side-length ε for each one of the D dimensions of the object to be covered, where c accounts for the 'length' in that dimension, and the ε-independent constant C is merely the product of the c's). From this scaling law, a linear relation is obtained:

$$\ln N(\varepsilon) = D \ln(1/\varepsilon) + \ln C, \tag{2}$$

and from it, definition (1) follows (the ε-independent, constant term becomes negligible as $\varepsilon \to 0$); it is also now clear, that the measurement of the fractal dimension involves details from a wide range of scales (all those covered while varying ε). From the linear relation (2), one then sees that its ordinate can be interpreted as the natural logarithm of the number of pixels of the original image (in practice, the grid can not be made smaller than a pixel when $\varepsilon \to 0$). In our measurements the side-size was varied from a single pixel (0.05 *mm*) to the whole image size (1200 *pixels* in average).

2.1. Measurements

The specimens were evenly illuminated with white light and directly photographed with a digital 2.1 *MP* camera (Sony Cybershot DSC-P50). The resolution of the digital images is 72 *dpi*. It is worth mentioning that even though the contour of the black background in the wings of butterflies is well defined when looking at the whole structure, there is a gradual evanishing of its intensity at smaller scales. This requires a threshold analysis [25] to confirm that there is no meaningful change in the fractal dimension of the black background when this characteristic is taken into account; for our images with 256 different intensities of gray, most of the black background of the butterflies is found at the 100th gray intensity. A careful digital process that involves intensity-filtering, contrast-magnification, and background-noise elimination (described in [6]) was performed to each single photograph resulting in a significantly increased contour-definition image (some examples are shown in Figs. 1 and 3). Fractal dimension measurements were performed on these processed images, and the uncertainty due to the evanishing contours was included in the total uncertainty value quoted (see next subsection).

Only the fractal structure of the contour of the black portions in the wings is considered, this means the form of the wing edge, and of the network of wing veins; the minor differences in the tonality of the orange color in the wings does not seem to play a relevant rôle in distinguishing one (sub)species from another since such subtle color differences are similar to differences amongst individual members of the same (sub)species. One must also bear in mind that the fractal dimension measurement includes information from a very wide range of scales, from the tiniest details to the whole object. Therefore, in order to test how distinctive the fractal dimension may be, and to propose it as a way for differentiating groups of butterflies, the same measuring procedure was applied to butterflies from different, unrelated species. The simplest case is the *Eurema albula* (classified by Cramer in 1776), a completely white butterfly found from the south of Texas to Brasil [10,19], and which also allows to confirm the effectiveness of the imaging process and measuring code. A second case is the female of the *Neophasia terlootii* found in south Arizona, Sonora, Chihuahua, Sinaloa, Durango, and Nuevo León; their black geometrical structure and orange color look very similar to those of the monarch butterfly and there are not any detectable differences at first sight (Fig. 3, [9,24]). The second group of mimicry related butterflies that we have studied comprises the *Greta nero*

nero, and *Hypoleria cassotis* butterflies (classified by Hewitson in 1855 and H. W. Bates in 1864, respectively). The other studied species are: *Callicore pitheas*, *Dismorphia theucarila fortunata*, and *Aporia crataegi* (classified by Latreille in 1811, Lucas in 1854, and Linnaeus in 1758, respectively); examples are shown in Fig. 3.

2.2. Results

The obtained fractal dimension values are 1.632 ± 0.0036 for the monarch butterflies, 1.628 ± 0.0059 for the tropical viceroy, and 1.634 ± 0.0051 for the northern viceroy. The quoted uncertainties include the effects from the evanishing intensity at small scales, the standard deviation due to the measuring of several specimens for each (sub)species and the uncertainty in the fitting process of the slopes of the straight lines in Fig. 2. Our analysis was carried out using 10 specimens for the monarch butterflies, 6 for each one of the viceroys, and at least 4 for each one of the other (sub)species. Corresponding uncertainties due to the measuring of one specimen alone are: ± 0.025, ± 0.029, and ± 0.019 . As previously pointed out, our results then show no possible distinctions for the species and the two subspecies in terms of the fractal dimension of the geometrical black pattern in their wings. The only other possibility for distinguishing them is their distinctive orange color, which does not show a resemblance as striking as the fractal dimension. This possibility however, is ruled out by the fact that these differences in intensity and tonality are similar to the differences amongst individual specimens of the same (sub)species.

The fractal dimension values obtained for the other cases are presented in the table and plotted in Fig. 4, where it can be seen that the members of the *Greta-Hypoleria* mimicry ring do share a common fractal dimension value for their wings' patterns. These results also show that wing patterns which do look similar, do not have the same fractal dimension (consider the *Neophasia terlooti* and the Monarch-Viceroys mimicry ring, or the *Dismorphia Theucarila* and the *Greta-Hypoleria* mimicry ring); while some others whose appearance is not so similar, do share a common fractal dimension value (the *Lycorea ilione* and the Monarch-Viceroys mimicry ring).

3. Discussion

Two mimicry rings have been analyzed in terms of the geometrical, black pattern in the wings of their members and it has been found that they do have the same fractal dimension. This is regarded as an important fact, specially when one considers that this measurement includes information from a very wide range of scales, from the tiniest details to the whole object. It has also been shown that the measurement of the fractal dimension may be used to differentiate between species that look similar but do not belong to the same mimicry ring.

With respect to the Monarch-Viceroys mimicry ring, our analysis raises some questions. It is a well-known fact in a wide variety of species that colors are used as warning signs that indicate increasing levels of toxicity with color brightness. It is also known that birds, predators in the present case, are not color blind [11]. However, it must be emphasized that the subtle distinct tonalities in the bright orange color of the monarch butterflies with respect to any of their two viceroy impersonators are very similar to the differences found in members of the same (sub)species. Thus, even if predators were able to perceive these subtleties (something that we do not know for certain), they would not be tricked by the mimicry achieved by the viceroy butterflies (since such differences are also present within the same unpalatable subspecies), and therefore would still pose a considerable threat to the butterflies survival (ruling out predation as the acting agent in natural selection). Furthermore, in view of the migrating habits of the monarch butterflies, it would remain to be explained how this mimicry (either batesian or müllerian) could be established for a species (monarchs) and a subspecies (tropical viceroys) that stay in contact, and thus possibly exert influence on each other for only a certain lapse during each year. For a discussion on the lack of coevolution see [21].

It thus may seem more appropriate to assume that the common feature identified in the two mimicry rings through the fractal analysis of their wings' patterns is due to the sharing of a common set of developmental raw material and modified by natural and/or sexual selection; it would also look as if the selection agent that has been acting on them is predation [27]. This particular point is consistent with the constraints derived from the accepted rules in the wing development programme (Nymphalid Ground Plan, [2]) and the genetic evidence in favor of the existence of on-off networks of genes that limit the number of possible final states [5]. These findings also agree with other data suggesting that the characteristics that define species are often the products of natural selection; it is now possible to identify the precise genes that contribute to speciation and the forces that drove their evolution [26].

It may then well be that the measurement of the fractal dimension of the wings' patterns can be used to identify groups where the developmental raw material is a common feature only modified by natural selection [20].

We are aware that much further work is needed to broaden the basis on which our results stand, to confirm and, if this is the case, to extend them. Nonetheless, we think it necessary to make them widely known and to ask other scientists to help in the task, specially if it is found that they can really represent a new powerful tool for identifying groups where the developmental raw material is a common feature and thus, simplify the finding of case studies where this may occur. Some work in this direction is already under progress; in particular, we are looking for a mechanism to produce fractal patterns similar to those present in the analyzed wings, and also trying to identify a particular feature in the butterflies' habitats whose fractal analysis might show a correlation to the geometric patterns in the wings.

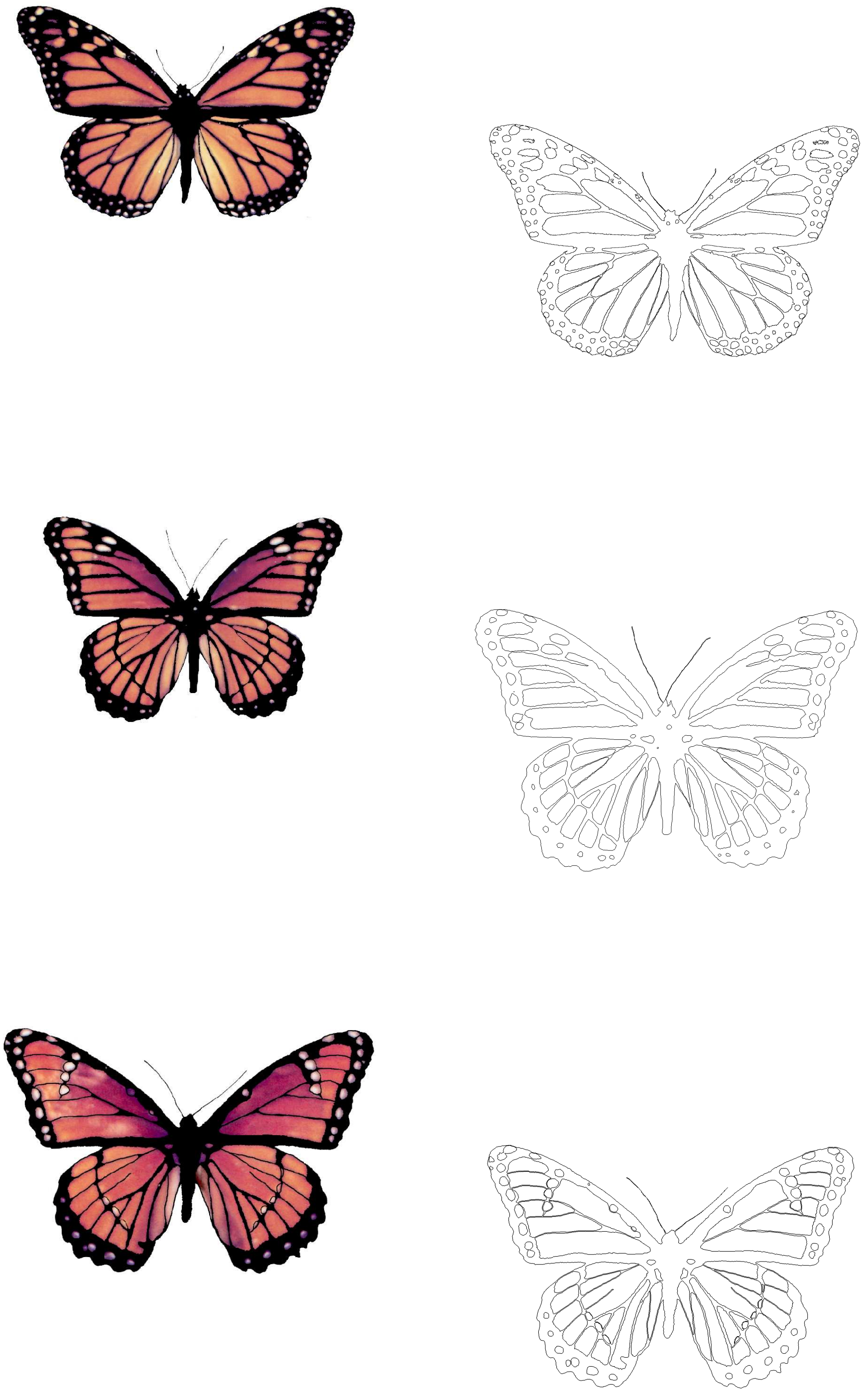

Fig. 1. Typical specimens of the *Danaus plexippus* or monarch butterfly, *Basilarchia archippus hoffmanni* or tropical viceroy, and *Basilarchia archippus obsoleta* or northern viceroy; their corresponding contour images are also shown.

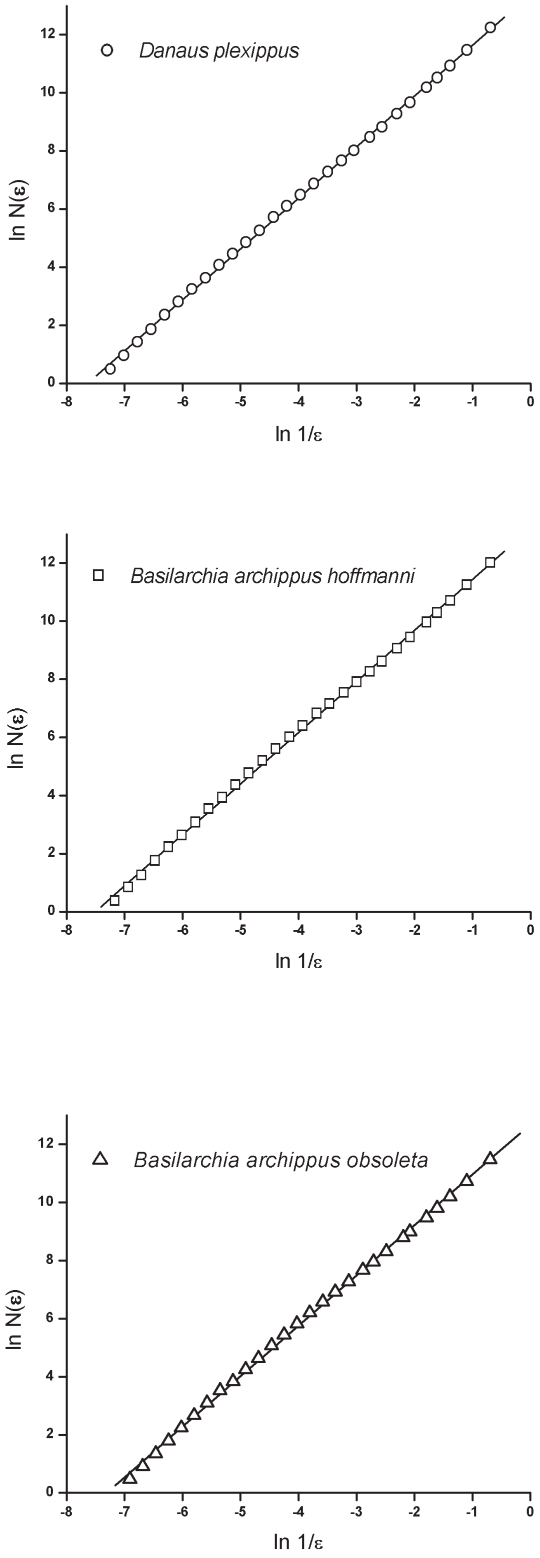

Fig. 2. Results of the box-counting method applied to the images in Fig. 1.

Fig. 3. Typical specimens of the *Lycorea ilione albescens*, *Greta nero*, *Hypoleria cassotis*, *Dismorphia theucarila fortunata*, *Aporia crataegi*, and the *Neophasia terlootii* female, and their corresponding contour images; the *Eurema albula*, and *Callicore pitheas* butterflies are not presented since their wings' patterns are unobservable or very simple.

Fractal dimension and corresponding uncertainty for the analyzed (sub)species; some of these values are plotted in Fig. 4 with the same numbering.

No.	Name	Dimension	Uncertainty
0	Eurema Albula	0.000	0.0000
1	Callicore pitheas	1.229	0.0127
2	Neophasia terlootii	1.478	0.0030
3	Greta nero	1.524	0.0028
4	Hypoleria cassotis	1.527	0.0099
5	Dismorphia theucarila fortunata	1.585	0.0057
6	Lycorea ilione albescens	1.609	0.0042
7	Basilarchia archippus hoffmanni	1.628	0.0059
8	Danaus plexippus	1.632	0.0036
9	Basilarchia archippus obsoleta	1.634	0.0051
10	Aporia crataegi	1.745	0.0057

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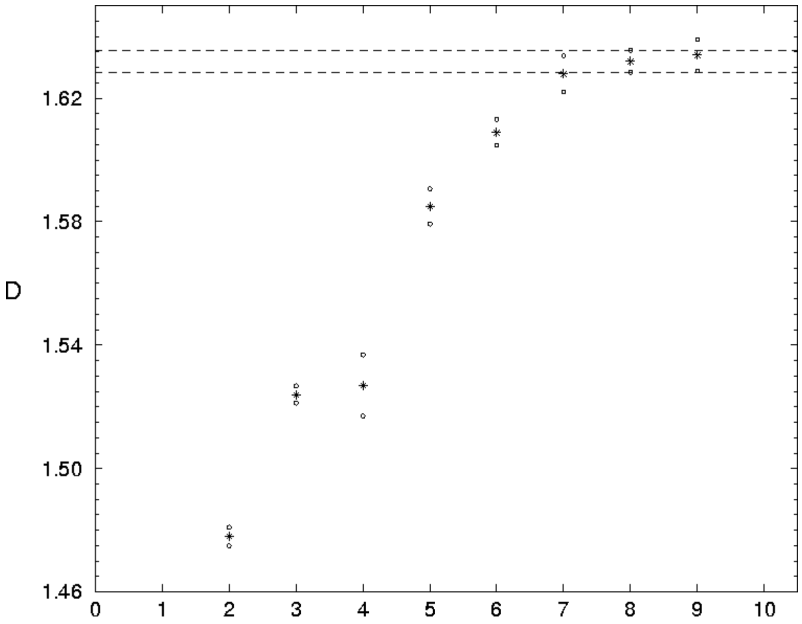

Fig. 4. Fractal dimension D of the wing patterns for the butterflies listed in the table. Horizontal lines indicate the uncertainties around the value for the *Danaus plexippus*. Data for the *Eurema albula*, *Callicore pitheas*, and *Aporia crataegi* butterflies have not been plotted in order to enlarge the graph in the vertical direction.

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