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Vacuum stress-energy tensor of the electromagnetic field in rotating frames

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A formalism for calculating the vacuum stress-energy tensor of the electromagnetic field in an arbitrarily moving frame is presented and applied to a system in uniform rotation. The spectra of the energy density, Poynting flux, and stress of the zero-point field are calculated in such a frame. A comparison is made with the case of linear acceleration and its associated thermal effects.

I. INTRODUCTION

It is now generally accepted that the zero-point fluctuations of a quantum field produce thermal-like effects in a gravitational field,[1] an accelerated system,[2] and, in general, a variety of other situations in which noninertial frames are involved.[3] It has been stressed by several authors[4–6] that the underlying mechanism of these effects is the fact that the energy spectrum of the zero-point field is Lorentz invariant, and, thus, it cannot be observed in an inertial frame, but manifests itself in noninertial frames.

While the theoretical aspects of the quantum fluctuations in gravitational fields or in accelerated frames are now well established, the experimental evidence for or against these interesting phenomena still remains unknown. The problem, of course, is that any vacuum-polarization effect is extremely tiny to be easily observed under usual laboratory conditions. For practical purposes, a uniformly rotating system may be the most appropriate frame for laboratory experiments: for instance, it has been suggested by Bell and Leinaas[7] that vacuum effects could be detected through the polarization of electrons circling in a storage ring at ultrarelativistic speeds (Lorentz factor $\gamma \sim 10^5$).

The aim of the present paper is to calculate explicitly the vacuum stress-energy tensor of the electromagnetic field as detected in a uniformly rotating frame. It is shown by numerical calculations that the energy density and stress have a spectrum which does not correspond to that of a system in thermal equilibrium, and that there is also an additional flux of energy given by a nonzero Poynting vector. These calculations generalize and complete the work of several previous authors who calculated the energy spectrum in a uniformly rotating system, both for a scalar massless field[8,9] and for the electromagnetic field.[10]

Section II of this paper deals with the derivation of the stress-energy tensor from the vacuum-expectation values of the electromagnetic field, generalizing a previously presented formalism[6,11] (which is also applicable to stochastic classical fields[4]). In Sec. III a brief review of a uniformly accelerated frame is given for the purposes of illustration and comparison. The stress-energy tensor in a rotating frame is calculated in Sec. IV and the results are discussed in Sec. V.

II. GENERAL THEORY

The electromagnetic energy-momentum tensor is defined by

$$T_{\mu\nu}=\frac{1}{16\pi}\langle 4F_{\mu\alpha}F^{\alpha}_{\nu}+\eta_{\mu\nu}F_{\lambda\beta}F^{\lambda\beta}\rangle \ , \tag{2.1}$$

where $\eta_{\mu\nu}$ is the Minkowski tensor and $F_{\mu\nu}$ is the electromagnetic field tensor which satisfies the vacuum Maxwell equations:

$$\partial_\mu F^{\mu\nu}=0 \ , \tag{2.2a}$$

$$\partial_{[\mu}F_{\nu\lambda]}=0 \tag{2.2b}$$

(hereafter the units employed are such that $c=1=\hbar$).

The form of the energy-momentum tensor suggests to define the two-point tensors

$$D^{+}_{\mu\nu}(x,x')\equiv\tfrac{1}{4}\langle 4F^{\alpha}_{(\mu}(x)F_{\nu)\alpha}(x')+\eta_{\mu\nu}F_{\lambda\beta}(x)F^{\lambda\beta}(x')\rangle \ , \tag{2.3a}$$

$$D^{-}_{\mu\nu}(x,x')\equiv D^{+}_{\mu\nu}(x',x) \tag{2.3b}$$

as generalizations of the Wightman functions used in the scalar case.[6]

It is easy to see that

$$\eta^{\mu\nu}D^{\pm}_{\mu\nu}=0 \ , \tag{2.4a}$$

$$D^{\pm}_{\mu\nu}=D^{\pm}_{\nu\mu} \ , \tag{2.4b}$$

and

$$\partial_\nu D^{\pm\nu}_{\mu}=0 \ , \tag{2.4c}$$

these last formulas being a consequence of the Maxwell equations.

Now, Eqs. (2.4) imply that

$$D^{\pm}_{\mu\nu}(x,x')=n\,\partial^2_{\mu\nu}D^{\pm}(x,x') \ , \tag{2.5}$$

where $D^{\pm}(x,x')$ are the usual Wightman functions for a massless scalar field, and n is some constant to be determined subsequently.

It is evident from the definitions (2.3) that the energy-momentum tensor $T_{\mu\nu}$ can be obtained by taking the coincidence limit of $D^{\pm}_{\mu\nu}(x,x')$ when $x'\to x$. However, some considerations are necessary since this limit is infinite.

Following the approach of our previous work,[6,11] we refer all measurable quantities to a given observer with world line $x^\alpha = x^\alpha(\tau)$ and four-velocity $u^\alpha = dx^\alpha/d\tau$, τ being his proper time. The energy-momentum tensor locally detected by this particular observer can be written as

$$T_{\mu\nu}[x^\alpha(\tau)] = \frac{1}{16\pi}\int_{-\infty}^{\infty}\delta(\sigma)\times\langle 4F^\alpha_{(\mu}(\tau+\sigma/2)F_{\nu)\alpha}(\tau-\sigma/2) + \eta_{\mu\nu}F_{\lambda\beta}(\tau+\sigma/2)\times F^{\lambda\beta}(\tau-\sigma/2)\rangle d\sigma\ , \tag{2.6}$$

where it is understood that

$$F_{\mu\nu}(\tau\pm\sigma/2) \equiv F_{\mu\nu}[x^\alpha(\tau\pm\sigma/2)]\ . \tag{2.7}$$

Using now the representation

$$\delta(\sigma) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\omega\sigma}d\omega \tag{2.8}$$

for the δ function, Eq. (2.6) can be written as

$$T_{\mu\nu}[x^\alpha(\tau)] = \frac{1}{8\pi^2}\times\int_{-\infty}^{\infty}\int_0^{\infty} e^{i\omega\sigma}[D^+_{\mu\nu}(\tau+\sigma/2,\tau-\sigma/2) + D^-_{\mu\nu}(\tau+\sigma/2,\tau-\sigma/2)]\times d\omega\, d\sigma\ . \tag{2.9}$$

Furthermore, using the Fourier transform

$$\tilde{D}^{\pm}_{\mu\nu}(\tau,\omega) = \int_{-\infty}^{\infty} e^{i\omega\sigma}D^{\pm}_{\mu\nu}(\tau+\sigma/2,\tau-\sigma/2)d\sigma\ , \tag{2.10}$$

the energy-momentum tensor takes the form

$$T_{\mu\nu}[x^\alpha(\tau)] = \frac{1}{8\pi^2}\int_0^{\infty}[\tilde{D}^+_{\mu\nu}(\tau,\omega)+\tilde{D}^-_{\mu\nu}(\tau,\omega)]d\omega\ , \tag{2.11}$$

which explicitly exhibits the energy spectrum in terms of the frequency ω associated directly with the observer's proper time τ.

On the other hand, the Wightman functions $D^{\pm}(x,x')$ for flat spacetime without boundary conditions (the only case considered in this paper) are proportional to

$$[(t-t'\mp i\epsilon)^2 - |\mathbf{x}-\mathbf{x}'|^2]^{-1}$$

and, therefore, according to Eq. (2.5),

$$D^{\pm}_{\mu\nu}(x,x') = \frac{4}{\pi}\frac{4(x_\mu-x'_\mu)(x_\nu-x'_\nu) - \eta_{\mu\nu}(x_\alpha-x'_\alpha)(x^\alpha-x'^\alpha)}{[(t-t'\mp i\epsilon)^2-|\mathbf{x}-\mathbf{x}'|^2]^3}\ , \tag{2.12}$$

where a factor of $16/\pi$ has been included in anticipation of the following results.

It is worth noticing that the function $D^{\pm}_{\mu\nu}$ has in general the form

$$D^{\pm}_{\mu\nu}(\tau+\sigma/2,\tau-\sigma/2) = \frac{A_{\mu\nu}}{(\sigma-i\epsilon)^4} - \frac{B_{\mu\nu}}{(\sigma-i\epsilon)^2} + D_{\mu\nu}(\tau,\sigma)\ , \tag{2.13}$$

where $A_{\mu\nu}$ and $B_{\mu\nu}$ depend only on τ, and $D_{\mu\nu}(\tau,\sigma)$ is by definition free of poles at $\sigma = \pm i\epsilon$.

In order to obtain the total stress-energy tensor, one has to integrate over the frequency ω. This integration can be done directly by using the formula for the inverse Fourier transform. The final result is

$$T_{\mu\nu}[u^\alpha(\tau)] = \frac{1}{8\pi}\int_0^{\infty}\left[\frac{A_{\mu\nu}}{3}\omega^3 + 2B_{\mu\nu}\omega\right]d\omega + \frac{1}{4\pi}D_{\mu\nu}(\tau,0)\ , \tag{2.14}$$

where the divergent integral corresponds to the zero-point energy, and the last term gives the physically observable stress-energy tensor.

III. LINEAR ACCELERATION

As a simple application of the formalism presented above, let us consider the well-known case of a uniformly accelerated observer whose world line and four-velocity are given by the parametric equations

$$x^\alpha(\tau) = a^{-1}[\sinh(a\tau),\cosh(a\tau),0,0]\ , \tag{3.1a}$$

$$u^\alpha(\tau) = [\cosh(a\tau),\sinh(a\tau),0,0]\ , \tag{3.1b}$$

where a is the linear acceleration. Therefore,

$$x^\alpha(\tau+\sigma/2) - x^\alpha(\tau-\sigma/2) = 2a^{-1}\sinh(a\sigma/2)u^\alpha(\tau)\ , \tag{3.2}$$

and, according to Eq. (2.12),

$$D^{\pm}_{\mu\nu}(\tau+\sigma/2,\tau-\sigma/2) = \frac{a^4\operatorname{csch}^4[a(\sigma\mp i\epsilon)/2]}{4\pi}\times[4u_\mu(\tau)u_\nu(\tau)-\eta_{\mu\nu}]\ . \tag{3.3}$$

Taking Fourier transforms,[11] one obtains the final result

$$T_{\mu\nu} = \frac{1}{3\pi^2}(4u_\mu u_\nu - \eta_{\mu\nu})\times\int_0^{\infty}\omega(\omega^2+a^2)\left[\frac{1}{2}+\frac{1}{e^{2\pi\omega/a}-1}\right]d\omega\ . \tag{3.4}$$

Clearly, $T_{\mu\nu}$ is proportional to $4u_\mu u_\nu - \eta_{\mu\nu}$, the only available traceless tensor. Furthermore, the integral exhibits the well-known (strictly infinite) zero-point energy spectrum, and an additional Planckian distribution combined with a modified density of states[11] $(\omega^2+a^2)d\omega$.

The energy-momentum four-vector is

$$J_\mu = T_{\mu\nu}u^\nu \equiv e u_\mu \ , \tag{3.5}$$

where e is the energy density. It is evident that the Poynting four-vector

$$(\delta^\mu_\nu - u^\mu u_\nu)J^\nu$$

vanishes: a uniformly accelerated observer detects no flux of electromagnetic energy. Notice also that the normalization of the Wightman tensor in Eq. (2.12) has been chosen in such a way that the energy per unit frequency $de/d\omega$ becomes equal to the usual value $\omega^3/2\pi^2$ in the limit of zero acceleration.

The total energy density e can be evaluated by subtracting the zero-point term before performing the integration in Eq. (3.4). The result is

$$e = \frac{11}{240\pi^2}a^4 \ , \tag{3.6}$$

the main contribution comes from the term proportional to a^2; the energy density for a scalar field is a factor of 11 smaller than the value given by Eq. (3.6).

IV. UNIFORM ROTATION

A uniformly rotating observer whose proper time is τ and angular speed is Ω follows a world line given by

$$x^\alpha = (\gamma\tau, R_0\cos(\Omega\tau), R_0\sin(\Omega\tau), \text{const}) \ , \tag{4.1}$$

where R_0 is the rotation radius in the inertial frame and $\gamma=(1-v^2)^{-1/2}$. The four-velocity $dx^\alpha/d\tau$ is then

$$u^\alpha = (\gamma, -\gamma v\sin(\Omega\tau), \gamma v\cos(\Omega\tau), 0) \ , \tag{4.2}$$

where $\gamma v = \Omega R_0$.

For this case, there are two Killing vectors:

$$k^\alpha = (1,0,0,0) \tag{4.3a}$$

and

$$m^\alpha = (0, -R_0\sin(\Omega\tau), R_0\cos(\Omega\tau), 0) \ , \tag{4.3b}$$

such that

$$k^\alpha k_\alpha = 1, \quad m^\alpha m_\alpha = -R_0^2, \quad \text{and } m^\alpha k_\alpha = 0 \ . \tag{4.4}$$

Using Eqs. (4.3), u^α can be rewritten as

$$u^\alpha(\tau) = \gamma k^\alpha + \Omega m^\alpha(\tau) \ , \tag{4.5}$$

whereby

$$u^\alpha k_\alpha = \gamma \quad \text{and } u^\alpha m_\alpha = -\Omega R_0^2 = (1-\gamma^2)/\Omega \ . \tag{4.6}$$

In terms of the Killing vectors [Eqs. (4.3)], we get

$$x^\alpha(\tau+\sigma/2) - x^\alpha(\tau-\sigma/2) = \gamma\sigma k^\alpha + 2\sin(\Omega\sigma/2)m^\alpha(\tau) \tag{4.7}$$

and

$$D^\pm_{\mu\nu}(\sigma) = \frac{8}{\pi}[4A^\pm k_\mu k_\nu + 4B^\pm(k_\mu m_\nu + m_\mu k_\nu) + 4(A^\pm C^\pm)m_\mu m_\nu/R_0^2 - C^\pm\eta_{\mu\nu}] \ , \tag{4.8}$$

where

$$A^\pm(\sigma) = \frac{\gamma^2\sigma^2}{z_\pm^3} \ , \tag{4.9a}$$

$$B^\pm(\sigma) = \frac{2\gamma\sigma\sin(\Omega\sigma/2)}{z_\pm^3} \ , \tag{4.9b}$$

$$C^\pm(\sigma) = \frac{1}{z_\pm^2} \ , \tag{4.9c}$$

and

$$z_\pm = \gamma^2(\sigma \mp i\epsilon)^2 - 4R_0^2\sin^2(\Omega\sigma/2 \mp i\epsilon) \ . \tag{4.9d}$$

According to the formalism in Sec. II, the electromagnetic energy-momentum tensor is given by

$$T_{\mu\nu} = \frac{1}{8\pi^2}\int_0^\infty [\tilde{D}^+_{\mu\nu}(\omega) + \tilde{D}^-_{\mu\nu}(\omega)]d\omega \ , \tag{4.10}$$

where

$$\tilde{D}^\pm_{\mu\nu}(\omega) = \int_{-\infty}^\infty D^\pm_{\mu\nu}(\sigma)e^{i\omega\sigma}d\sigma \ , \tag{4.11}$$

and the energy-momentum four-vector by

$$J_\mu \equiv u^\nu T_{\mu\nu} = e u_\mu + p n_\mu \ , \tag{4.12}$$

where $e = u_\mu u_\nu T_{\mu\nu}$ is the energy density of the field, pn_α is the Poynting four-vector, and

$$n_\alpha \equiv \frac{m_\alpha}{\gamma^2 R} + v u_\alpha \tag{4.13}$$

is a unit vector in the direction of the Killing vector m^α projected on the space of the rotating observer. Clearly

$$n^\alpha u_\alpha = 0, \quad m^\alpha = \gamma R_0(n^\alpha - v u^\alpha), \quad \text{and } k^\alpha = \gamma(u^\alpha - v n^\alpha) \ . \tag{4.14}$$

Using Eqs. (4.8) and (4.14), we get from Eq. (4.10) that

$$T_{\mu\nu} = e u_\mu u_\nu + p(u_\mu n_\nu + n_\mu u_\nu) + s(u_\mu u_\nu - \eta_{\mu\nu}) + (e-3s)n_\mu n_\nu \ , \tag{4.15}$$

where s is the magnitude of the electromagnetic stress.

Inserting Eq. (4.15) in Eq. (4.12), we get

$$\frac{de}{d\omega} = \frac{\gamma^2}{\pi^3}[4(1+v^2)(\tilde{A}^+ + \tilde{A}^-) - 8vR_0(\tilde{B}^+ + \tilde{B}^-) - (1+3v^2)(\tilde{C}^+ + \tilde{C}^-)] \ , \tag{4.16a}$$

$$\frac{dp}{d\omega} = -\frac{4\gamma R_0}{\pi^3}[2\Omega(\tilde{A}^+ + \tilde{A}^-) - \gamma(1+v^2)(\tilde{B}^+ \tilde{B}^-) - \Omega(\tilde{C}^+ + \tilde{C}^-)] \ , \tag{4.16b}$$

and

$$\frac{ds}{d\omega} = \frac{1}{\pi^3}(\tilde{C}^+ + \tilde{C}^-) \ , \tag{4.16c}$$

where a tilde denotes the Fourier transform [according to Eq. (4.11)] of the corresponding function.

It is important to note that, since the energy flux detected by the rotating observer is in the direction of the tangential velocity, *the observed radiation is not isotropic.*

Equations (4.9) can be rewritten as (see the Appendix)

$$A^{\pm}(\sigma)=\gamma^2\left[\frac{1}{(\sigma\mp i\epsilon)^4}-\frac{(\gamma v\Omega)^2}{4(\sigma\mp i\epsilon)^2}\right]+A(\sigma)\ , \qquad (4.17a)$$

$$B^{\pm}(\sigma)=\gamma\Omega\left[\frac{1}{(\sigma\mp i\epsilon)^4}-\frac{(\gamma^2-\frac{5}{6})\Omega^2}{4(\sigma\mp i\epsilon)^2}\right]+B(\sigma)\ , \qquad (4.17b)$$

and

$$C^{\pm}(\sigma)=\frac{1}{(\sigma\mp i\epsilon)^4}-\frac{(\gamma v\Omega)^2}{6(\sigma\mp i\epsilon)^2}+C(\sigma)\ , \qquad (4.17c)$$

where $A(\sigma)$, $B(\sigma)$, and $C(\sigma)$ are defined by Eq. (4.17) and are free of poles near the origin at $\sigma=\pm i\epsilon$.

The Fourier transforms of the functions $A^{\pm}$, $B^{\pm}$, and $C^{\pm}$ are now given by

$$\tilde{A}^{+}(\omega)=\pi\gamma^2\omega\left[\frac{\omega^2}{6}+\frac{\Omega^2}{4}(\gamma^2-1)\right]+\tilde{A}(\omega)\ ,\quad \tilde{A}^{-}(\omega)=\tilde{A}(\omega)\ , \qquad (4.18a)$$

$$\tilde{B}^{+}(\omega)=\pi\gamma\Omega\omega\left[\frac{\omega^2}{6}+\frac{\Omega^2}{4}(\gamma^2-\tfrac{5}{6})\right]+\tilde{B}(\omega)\ ,\quad \tilde{B}^{-}(\omega)=\tilde{B}(\omega)\ , \qquad (4.18b)$$

and

$$\tilde{C}^{+}(\omega)=\frac{\pi\omega}{6}[\omega^2+\Omega^2(\gamma^2-1)]+\tilde{C}(\omega)\ ,\quad \tilde{C}^{-}(\omega)=\tilde{C}(\omega)\ , \qquad (4.18c)$$

which inserted in Eqs. (4.16) give

$$\frac{de}{d\omega}=\frac{\omega}{2\pi^2}[\omega^2+(\gamma v\Omega)^2]+\frac{2\gamma^2}{\pi^3}[4(1+v^2)\tilde{A}(\omega)-8vR_0\tilde{B}(\omega)-(1+3v^2)\tilde{C}(\omega)]\ , \qquad (4.19a)$$

$$\frac{dp}{d\omega}=\frac{\gamma^2v\Omega^2\omega}{6\pi^2}-\frac{8\gamma R_0}{\pi^3}[2\Omega\tilde{A}(\omega)-\gamma(1+v^2)\tilde{B}(\omega)-\Omega\tilde{C}(\omega)]\ , \qquad (4.19b)$$

and

$$\frac{ds}{d\omega}=\frac{\omega}{6\pi^2}[\omega^2+(\gamma v\Omega)^2]+\frac{2\tilde{C}(\omega)}{\pi^3}\ . \qquad (4.19c)$$

The zero-point field contributes to the energy density, energy flux, and stress of the electromagnetic field, as detected by a rotating observer, in different ways. It contributes to the energy density [Eqs. (4.19a)] with the term $[\omega^2+(\gamma v\Omega)^2]\omega/2\pi^2$ due to a modification of the density of states, just as in the case of a linear acceleration[10] with $a=\gamma v\Omega$; its contribution to the energy flux [Eq. (4.19b)] is $\Omega^2\gamma^2v\omega/6\pi^2$; and it contributes to the stress [Eq. (4.19c)] with one-third of the energy-density contribution.

The divergent stress-energy tensor due to the zero-point field is then given by

$$T_{\mu\nu}^{(\infty)}=\frac{2}{3\pi^2}\int_0^{\infty}[\omega^2+(\Omega\gamma v)^2]\omega\,d\omega(u_\mu u_\nu-\tfrac{1}{4}\eta_{\mu\nu})+\frac{\Omega^2\gamma^2v}{6\pi^2}\int_0^{\infty}\omega\,d\omega(u_\mu n_\nu+n_\mu u_\nu)\ . \qquad (4.20)$$

The first term could have been guessed by a simple comparison with the similar equation for linear acceleration [Eq. (3.4)]; the second term, however, is entirely due to the rotation and implies a Poynting flux of energy.

Subtracting the zero-point field contributions from Eqs. (4.19) one obtains the physically observable terms

$$\frac{de}{d\omega}=\frac{\gamma^3}{2\pi^3R_0^3}\frac{\omega^2+(\gamma v\Omega)^2}{\omega^2}H_\gamma(2\omega/\Omega)\ , \qquad (4.21a)$$

$$\frac{dp}{d\omega}=\frac{\gamma^3}{2\pi^3R_0^3}\frac{\omega^2+(\gamma v\Omega)^2}{\omega^2}K_\gamma(2\omega/\Omega)\ , \qquad (4.21b)$$

and

$$\frac{ds}{d\omega}=\frac{\gamma^3}{2\pi^3R_0^3}\frac{\omega^2+(\gamma v\Omega)^2}{\omega^2}J_\gamma(2\omega/\Omega)\ , \qquad (4.21c)$$

where

$$H_\gamma(w)\equiv\frac{v^3w^2}{w^2+(2\gamma v)^2}\int_0^{\infty}\left[\frac{(3+v^2)x^2+(1+3v^2)v^2\sin^2x-8v^2x\sin x}{\gamma^2[x^2-v^2\sin^2x]^3}-\frac{3}{x^4}+\frac{2\gamma^2v^2}{x^2}\right]\cos(wx)dx\ , \qquad (4.22a)$$

$$K_\gamma(w)\equiv-4v^4\int_0^{\infty}\left[\frac{x^2+v^2\sin^2x-(1+v^2)x\sin x}{\gamma^2[x^2-v^2\sin^2x]^3}-\frac{\gamma^2}{6x^2}\right]\cos(wx)dx\ , \qquad (4.22b)$$

$$J_\gamma(w)\equiv\frac{v^3w^2}{w^2+(2\gamma v)^2}\int_0^{\infty}\left[\frac{1}{\gamma^4[x^2-v^2\sin^2x]^2}-\frac{1}{x^4}+\frac{2\gamma^2v^2}{3x^2}\right]\cos(wx)dx\ ; \qquad (4.22c)$$

the factor $\omega^2/[\omega^2+(\gamma v\Omega)^2]$ has been included in Eqs. (4.21) in order to reproduce functions similar to Planckian distributions and facilitate a comparison with the case of linear acceleration.

In order to evaluate the integrals in Eqs. (4.22) it is most convenient to notice that, in the ultrarelativistic limit $\gamma\gg1$, the terms within the square brackets in these integrals are proportional to γ^4 times a function of γx.

Such a scaling property can be checked by a series expansion or by numerical calculation, the error being of order γ^{-2}. Thus, for large γ we have $H_{k\gamma}(kw)=k^3H_\gamma(w)$, where k is an arbitrary constant, and similarly for $K_\gamma(w)$ and $J_\gamma(w)$; notice that this is the same scaling property of a Planckian distribution with a temperature proportional to γ. The graphs of these functions are shown in Fig. 1. Equation (4.21) was previously found and numerically integrated in Ref. 10, both for nonrelativistic and relativistic velocities with $\gamma<10$.

The total electromagnetic energy density, energy flux, and stress can be obtained integrating Eqs. (4.19) (see the Appendix):

$$e=\frac{\gamma^4\Omega^4 v^2}{360\pi^2}(50-33\gamma^{-2})\,, \tag{4.23a}$$

$$p=\frac{\gamma^4\Omega^4 v}{720\pi^2}(50-47\gamma^{-2})\,, \tag{4.23b}$$

$$s=\frac{\gamma^4\Omega^4 v^2}{360\pi^2}(11-15\gamma^{-2})\,. \tag{4.23c}$$

At this point it is tempting to compare the cases of uniform acceleration and circular motion, in order to fit the spectra of Fig. 1 by Planckian functions. Since one may identify the acceleration a with $\gamma v\Omega$, one would intuitively believe that the two cases are quite similar. However, it is enough to compare Eqs. (3.6) and (4.23a) for the energy density of both cases to see that there are important differences: to begin with, the formula for the linear acceleration predicts an energy density which does not numerically coincide with the equivalent formula for circular motion; even worse is the fact that e is of order $(\gamma v\Omega)^4$ in the first case, whereas it is of order $(\gamma\Omega)^2v^2$ in the second one [Eqs. (3.6) and (4.23a)]. Two conclusions follow: first, it is only in the ultrarelativistic case $v\sim c$, that it makes some sense to compare, at least roughly, linear acceleration and circular motion; second, for nonrelativistic speeds and for a given value $\gamma v\Omega$ of the acceleration (either centrifugal or linear), uniform acceleration is less effective by a factor $(v/c)^2$ than circular motion in revealing the zero-point energy.

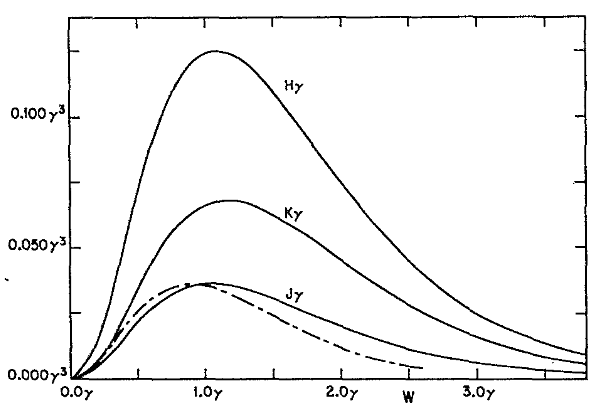

FIG. 1. The spectral functions $H_\gamma(w)$, $J_\gamma(w)$, and $K_\gamma(w)$. For ultrarelativistic velocities these functions can be expressed in terms of three universal functions due to their scaling property. The Planckian function (dash-dotted line) with temperature $\gamma/2\pi$ is plotted for comparison [see Eq. (4.24)].

Just for comparison, let us write the energy density in a frame with linear acceleration $\gamma v\Omega$ in a way similar to Eq. (4.21a):

$$e=\frac{\gamma^3}{2\pi^3R_0^3}\frac{\omega^2+(\gamma v\Omega)^2}{\omega^2}\left[\frac{\pi v^3}{4}\frac{w^3}{e^{2\pi w/\gamma v}-1}\right]; \tag{4.24}$$

it is the term in large parentheses which must be directly compared with $H_\gamma(w)$. This Planckian function is plotted in Fig. 1 for $\gamma\gg1$; clearly, it does not coincide with the spectral function $H_\gamma(w)$.

V. DISCUSSION

From the results obtained in the previous section it is clear that uniformly rotating frames exhibit some features which are absent in the well-studied case of uniformly accelerated systems. It has been shown in the preceding lines that the energy spectrum of the zero-point field, as detected by a rotating observer, acquires some additional terms which are not Planckian distributions, in contrast with the linearly accelerating case where the observed spectrum is strictly thermal (though with a modified density of states). Another interesting feature is the existence of a net flux of energy in the direction of motion of the rotating observer which is of order $\gamma^4\Omega^4 v$. If this flux is real, it should imply some friction-like effect on a rotating particle.

The formalism of this paper keeps track of the zero-point energy and does not discard it as is usually done in a similar analysis. This is why we obtain a full stress-energy tensor containing terms proportional to ω^3 and ω, which diverge upon integration over the frequency. Such terms can be eliminated if advanced Green's functions with poles slightly below the real axis are used instead of Wightman functions. We feel, however, that this is an *ad hoc* procedure and have decided to keep the zero-point energy as it follows directly from the formalism. This is in agreement with the previously expressed viewpoint that the thermal-like effects in noninertial frames are due to the Doppler-type distortions of the zero-point energy spectrum and are of the same nature as this field.[4,6]

Also, it is evident from the previous results that a simple subtraction of the zero-point field does not eliminate the diverging terms in the stress-energy tensor. Indeed, there are terms proportional to $\omega\,d\omega$ which appear even in a uniformly accelerated frame [Eq. (3.4)] and come from the modified density-of-states factor ω^2+s^2/a^2, where s is the spin of the field. The interesting fact is that terms proportional to ω contribute also to a Poynting flux in a rotating frame. It is not clear how to eliminate this diverging contribution unless one arbitrarily cuts all the infinite terms by appropriately moving the poles in the propagator of the field, as mentioned above. This is a point which should be clarified if one insists in assigning a physical reality to the zero-point field; it is no longer valid to claim that this field is not directly detectable because of its Lorentz invariance, since we have shown that it should produce some noticeable effects in a rotating system.

APPENDIX

Using the approximations given by

$$\frac{x^2}{[x^2-4v^2\Omega^{-2}\sin^2(\Omega x/2)]^3} = \frac{\gamma^6}{x^4}\left[1-\frac{(\gamma v\Omega)^2}{4}x^2+\frac{\gamma^2v^2\Omega^4}{24}(\gamma^2-\tfrac{4}{5})x^4 + O(x^6)\right], \tag{A1a}$$

$$\frac{x\sin(\Omega x/2)}{[x^2-4v^2\Omega^{-2}\sin^2(\Omega x/2)]^3} = \frac{\gamma^6\Omega}{2x^4}\left[1-\frac{\Omega^2}{4}(\gamma^2-\tfrac{5}{6})x^2 + \frac{\Omega^4}{24}\left[\gamma^4-\frac{31\gamma^2}{20}+\frac{9}{16}\right]x^4+O(x^6)\right], \tag{A1b}$$

and

$$\frac{1}{[x^2-4v^2\Omega^{-2}\sin^2(\Omega x/2)]^2} = \frac{\gamma^4}{x^4}\left[1-\frac{(\gamma v\Omega)^2}{6}x^2+\frac{\gamma^2v^2\Omega^4}{48}(\gamma^2-\tfrac{11}{15})x^4+O(x^6)\right] \tag{A1c}$$

it is easy to arrive at Eq. (4.17).

From Eqs. (A1) it is straightforward to obtain

$$A(0)=\frac{\gamma^6v^2\Omega^4}{120}(1+4v^2)\,, \tag{A2a}$$

$$B(0)=\frac{\gamma^5\Omega^5}{1920}(1+34v^2+45v^4)\,, \tag{A2b}$$

and

$$C(0)=\frac{\gamma^4v^2\Omega^4}{720}(4+11v^2)\,, \tag{A2c}$$

which are used for the calculation of the total electromagnetic energy density, energy flux, and stress.[6]

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