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Mass Function for Massive Stars

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Summary. H–R diagrams for selected regions in the galaxy are obtained from the catalogue of supergiant and O stars in associations and clusters (Humphreys, 1978). These diagrams are then normalized using the O stars densities given in the catalogue of galactic O stars (Cruz-González et al., 1974) and used to derive an initial and a present day mass functions for a particular region chosen as representative of the solar neighbourhood.

An extension of the H-burning stage to include B0 to B3 stars is proposed and found appropriate to remove the discrepancy between the observed number of Wolf-Rayet stars and the numbers obtained from previous estimates of the mass function for their possible progenitors (stars with $M \geqq 20 M_\odot$). Another discrepancy remains still unsolved: taking $\log T_{\rm eff} = 4.3$, as the lower temperature limit to the H-burning phase and using $M_{\rm bol} = -7$ as the upper luminosity limit to the same phase, the percentage of He-burning stars in the sample is found to be 20. This problem is thought to be related to the anomalous associations where the number of stars in the He-burning regions of the H–R diagram overpasses that of the stars in the main sequence band.

Finally, a comparison with previously published mass functions is presented and an attempt is made in order to explain the differences found therein.

Key words: galactic structure – mass function – massive stars – H–R diagram – stellar evolution.

I. Introduction

During the last decade several present-day mass functions (PDMFs) have been estimated (Lequeux, 1979; Miller and Scalo, 1979 and references therein) with the particularity that in general either it is found difficult to extend the function to the large masses region or the error quoted is too large in this region and the numbers become unreliable. This is mainly due to the difference between the density of small-mass stars (whose number is statistically meaningful) and the density of large-mass stars (rather scarce in number) derived from the catalogues. Errors due to fluctuations in the distribution of stars may thus easily slip into the estimates of the PDMFs.

An immediate difficulty arises when one compares the density of stars with a mass greater than or equal to $20 M_\odot$ in the H-burning phase within the main sequence and in the supergiants region to be observed number of Wolf-Rayet and red supergiant stars assumed to represent the post-main sequence, He-burning evolutionary state (Firmani, 1982).

Two possible alternatives to account for this discrepancy are: (i) the densities of massive stars in the H-burning phase predicted by the known PDMFs for a reduced solar neighbourhood sample and commonly accepted as correct are not directly comparable to the observed densities of Wolf-Rayet and red supergiant stars due to the possible density fluctuations mentioned above; or (ii) not all the H-burning objects are considered when one restricts the counting process to the stars within the formal main sequence band. Both possibilities are discussed in the present work.

II. Method of Estimation

The catalogue of galactic O Stars (CGO) by Cruz-González et al. (1974) is considered to be a reliable sample for the required estimation. However, the lack of stars in this catalogue whose temperatures are inferior or equal to that corresponding to the spectral type B0 forbids an evaluation of the extent of the H-burning phase.

This deficiency can be overcome by the use of the catalogue of supergiants and O stars in associations and clusters (HC) of Humphreys (1978) which although lacking a definite limit in magnitude extends over well defined regions of space and hence provides a complete ($M_{\rm bol} \leqq -6$) H–R diagram for these regions in the large mass range.

A "theoretical" H–R diagram obtained by "normalizing" the HC diagram with the O stars density from the CGO seems then to be the best option to overcome these difficulties.

III. Sample

As mentioned before the HC constitutes the source from which the statistics reported in the present work are taken. The catalogue is considered to be complete in the following sense.

1. The ratio of the number of the earliest, more luminous stars (those whose spectral type is O7 or earlier) to the number of the comparatively less luminous, later O stars (O8s and O9s) for different values of the distance modulus corrected by the mean absorption (averaged over the association to which every O star belongs) does not show any significant variation from cluster to cluster. If present, this variation would correspond to a decrease in

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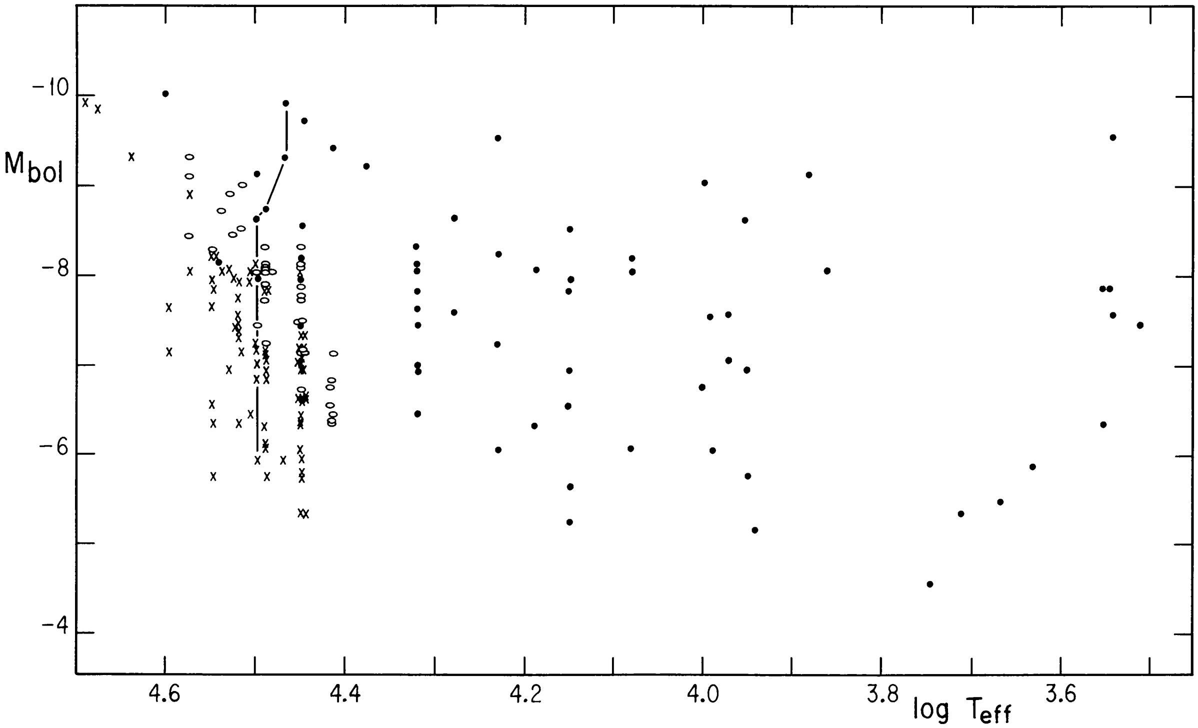

Fig. 1. H–R diagram from the stars in the solar neighbourhood group (previously defined). Dots denote supergiant stars, ovals represent giants and crosses indicate main sequence stars. The line goes through the latest O stars in the sample. The equivalence between spectral type and effective temperature was taken from Humphreys (1978)

the relative number of comparatively less luminous stars for the more distant associations and thus would show the degree of incompleteness.

2. When compared to some other catalogues such as the Michigan Spectral Catalogue (MSC) by Houk and Cowley (1975), the Catalogue of Bright Stars (CBS), by Hoffleit (1964) and the Catalogue of Galactic O stars (CGO) by Cruz-González et al. (1974) it was found that, each one of the associations in the HC contained approximately the same number of O stars as expected from the corresponding parts of the catalogues, given their completeness limits.

From the associations in HC three different groups have been considered: sample 1, henceforth called the solar neighbourhood, containing the stars within the associations in the local arm whose distance modulus corrected by the mean absorption is smaller than or equal to 13.5, i.e., SCT OB2, CYG OB3, CYG OB7, CEP OB2, CEP OB3, CAS OB14, CAM OB1, AUR OB1, GEM OB1, ORI OB1, MON OB2, CMA OB1, and VELA OB1; sample 2, made up by the stars within the associations having a distance modulus corrected by the mean absorption greater than 13.5 but smaller than or equal to 14.5 viz., SGR OB1, SGR OB7, SGR OB4, SGR OB6, SER OB1, SCT OB3, VUL OB1, CYG OB1, CYG OB9, CEP OB5, CAS OB5, CAS OB4, CAS OB1, NGC 457, CAS OB8, PER OB1, CAS OB6, AUR OB2, PUP OB1, CAR OB1, TR 14, TR 16, COLL 228, CAR OB2, NGC 3576, CRU OB1, CEN OB1, R 105, ARA OB1a, NGC 6204, and SCO OB1. The third sample used for comparison purposes only and being to some extent a subgroup of the second one, contains the members of CRU OB1, CEN OB1, R 103, ARA OB1a, and SCO OB1. The associations in Sample 3 were confronted with the MSC and those in the solar neighbourhood group with the CBS and CGO.

The H–R diagram of the stars within the associations in the solar neighbourhood group (Sample 1) is presented in Fig. 1.

IV. Statistics

In order to transform the number of stars within different effective temperature (or spectral type) intervals into densities one assumes the density of any such interval, σ, to be given by the product of the number of stars in the interval, N, times a "proportionality" or "normalizing" factor f, i.e., $\sigma = fN$. The factor f is obtained by combining the information in the CGO with that contained in the HC: the density of O stars given by the CGO is 19.40 stars kpc^{-2} up to 2.5 kpc and the number of O stars within the associations in the solar neighbourhood group is 56. The factor f for any spectral interval in Sample 1 is then given by $f_1 = 0.35\,\mathrm{kpc}^{-2}$. The CGO sample is reasonably complete up to a distance modulus of 12 (2.5 kpc) and none of the average distance moduli of the associations in Sample 1 is greater than 11.85. For the more distant Sample 2 the factor f is obtained using the stars earlier than or equal to O7.5. The CGO density for the later stellar group is 5.88 stars kpc^{-2} and there are 83 of them in Sample 2. The normalizing factor is therefore given by $f_2 = 0.07\,\mathrm{kpc}^{-2}$.

H–R diagrams made out of the stars within the associations in each sample (similar to that presented in Fig. 1) are divided in mass intervals using the evolutionary tracks of Chiosi et al. (1978) for the non-conservative case (Chiosi's parameters of mass loss $\alpha = 0.90$). This choice is not critical since mass loss does not significantly affect the M/L relation. The number of stars in each mass interval and the densities obtained in the way described above are presented in Table 1 for Samples 1 and 2 together with the adopted mean densities. These numbers include however, the stars in the He-burning phase which according to the evolutionary lifetimes of Chiosi et al. (1978) may at most be some 10% of each one of these numbers. Hence the densities of H-burning stars for the solar neighbourhood are accordingly calculated and presented in the last column of Table 1. The first one of these

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Table 1. Numbers of stars and densities calculated in the way described in the text

$M/M_\odot$	Sample 1		Sample 2		Adopted values (all spectral types)	H-burning stars (0–B3)
$\geqq$	No. of stars	σ Stars kpc^{-2}	No. of stars	σ Stars kpc^{-2}	σ (Mean densities) Stars kpc^{-2}	σ Stars kpc^{-2}
20	119	41.23	–	–	41	37
30	41	14.20	196	13.88	14	13
40	20	6.93	86	6.09	6	6
60	4	1.39	30	2.12	2	2

densities is compatible with the density of Wolf-Rayet stars (2WR stars kpc^{-2}) assuming a lifetime of roughly 5% of the total evolutionary lifetime, (Firmani, 1982) which are assumed to have the same original mass and to be burning He at present. They are compatible in the sense that the ratio of the Wolf-Rayet stars density to the H-burning stars density, which represents the ratio of the corresponding lifetimes, is ~0.05. The general contradiction between the density of Wolf-Rayet stars and the density of their probable ancestors obtained from the known PDMFs is thus solved.

A detailed analysis of the H–R diagram for the solar neighbourhood reveals an still unsolved problem. Figure 2 shows an histogram where the number of stars satisfying $-9 \leqq M_{\mathrm{bol}} \leqq -7$ is plotted against the logarithm of the effective temperature (T_{eff}). The histogram shows a severe decrease in the number of stars around $\log T_{\mathrm{eff}} = 4.3$ probably indicating the end of the H-burning evolutionary tracks which, if true, would mean an extension of the main sequence band to $T_{\mathrm{eff}} \sim 20{,}000$ K. However, the number of stars in the interval $\log T_{\mathrm{eff}} \leqq 4.3$ represents a 20% of the total in the sample which exceeds the expected number of stars in the He-burning phase ($\leqq 10\%$ according to Chiosi et al., 1978). A similar finding concerning the excess of stars outside the formal main sequence band has been recently published (Maeder, 1982). This problem has barely received attention and is probably related to the "anomalous" associations where the number of stars in this region of the H–R diagram surpasses that of the stars in the main sequence band; work in this direction is currently under progress.

V. Discussion

From the data in Table 1 an analytical expression can be derived for the integral present day mass function in the range $M \geqq 20\,M_\odot$. The resulting expression is given by

$$\sigma = 1.3\,10^5\,(M/M_\odot)^{-2.7}\,\mathrm{kpc}^{-2}. \tag{1}$$

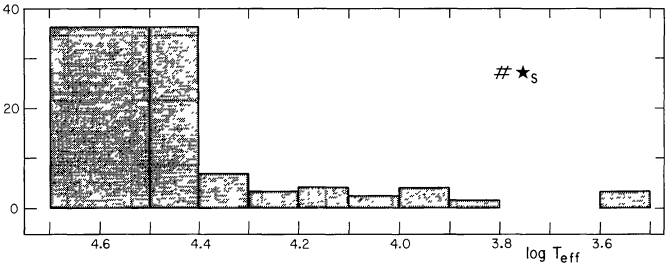

Fig. 2. Histogram showing the number of stars in the solar neighbourhood group with $-9 \leqq M_{\mathrm{bol}} \leqq -7$ as a function of the effective temperature

The only analytical expression for the main sequence lifetimes known to the authors is that given by Maeder (1981) as

$$t = 5.3\,10^7\,(M/M_\odot)^{-0.614}\,\mathrm{yr}, \tag{2}$$

this expression yields lifetimes which are practically equal to the ones obtained from the ($\alpha = 0.90$) evolutionary tracks of Chiosi et al. (1978). The initial mass function (IMF) can then be analytically expressed as

$$\frac{d\dot{\sigma}}{d\ln(M/M_\odot)} = -\frac{\sigma}{t}\,\frac{d\ln\sigma}{d\ln(M/M_\odot)} = 6.5\,10^{-3}\,(M/M_\odot)^{-2.1}\,\mathrm{yr}^{-1}\,\mathrm{kpc}^{-2} \tag{3}$$

or

$$\frac{d\dot{\sigma}}{d\log(M/M_\odot)} = 1.5\,10^{-2}\,(M/M_\odot)^{-2.1}\,\mathrm{yr}^{-1}\,\mathrm{kpc}^{-2}, \tag{4}$$

for the range $M \geqq 20\,M_\odot$.

The derived expression for the integral PDMF is a factor ~2.2 greater than its analogue calculated by Lequeux (1979) including the run-away stars and a factor ~1.6 greater than that derived by Miller and Scalo (1979) within the mass range $M \geqq 20\,M_\odot$.

Figure 3 shows the present day mass functions $\left(\phi = \frac{d\sigma}{dM}\right)$ derived by Miller and Scalo (1979) and by Lequeux (1979), and the one corresponding to the σ derived above. Extrapolating the latter it meets the function of Miller and Scalo at $\sim 10\,M_\odot$. The function and its exptrapolation are well within the error bars calculated by Miller and Scalo (1979) for their present day mass function.

As for the initial mass function a better agreement, when extrapolating, is found with the recent derivation of Tarrab (1982) for the mass interval $1.25 \leqq M/M_\odot \leqq 14$ using the available data on young open clusters. The slope of the IMF she finds is -1.7 whereas Miller and Scalo (1979) report ~ -1.5 for the slope in the mentioned mass interval.

A partial reason for the differences between the derived expression for the mass function and those found by Lequeux (1979) and Miller and Scalo (1979) is the extension of the H-burning stage to include B0 to B3 stars. This extension is thought to explain a factor of ~2 in the differences.

The use of different lifetime scales also means differences in the derived IMFs. This is the case when considering the IMF obtained by Miller and Scalo since the lifetimes adopted by them are smaller by a factor ~1.6 than those calculated with Maeder's expression, but it does not cause any significant difference for the IMF of Lequeux who uses the ($\alpha = 0.90$) lifetimes of Chiosi et al. (1978) that, as mentioned previously, are practically equal to the ones predicted by Maeder (1981).

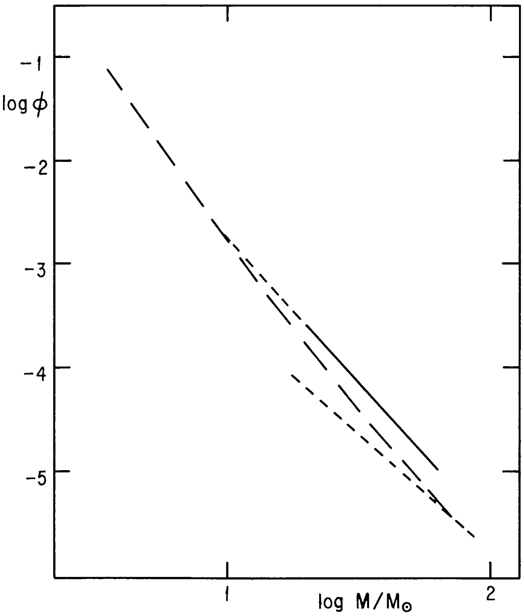

Fig. 3. The present day mass functions derived by Miller and Scalo (1979, long dashes), by Lequeux (1979, short dashes), and that derived following the method described in the text (solid line). An extrapolation of the latter to lower masses is shown by a dotted line

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Note: At the time of submitting this paper, a preprint of another on "The Initial Mass Function for Massive Stars" by Garmany et al. (1982) reached us. Their conclusions concerning the IMF are similar to the ones described above in the sense that their function is substantially higher than those of Lequeux (1979) and Miller and Scalo (1979) though they consider only O stars and their derivation is different in several aspects from the one being reported here. Their case *b* IMF (which is practically equal to their expression inside the solar circle) shows a varying difference with respect to ours, it is 1.1 smaller at $20\,M_\odot$ and 1.9 greater at $60\,M_\odot$.

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