Comment on "Acoustic chaos in a duct with two separate sound sources" [J. Acoust. Soc. Am. 110, 120–126 (2001)] (L)

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(Received 24 March 2008; revised 21 July 2008; accepted 30 July 2008)

In a paper published in this journal in 2001 by Dong *et al.* [W. G. Dong, X. Y. Huang, and Q. L. Wo, J. Acoust. Soc. Am. **110**, 120–126 (2001)] it was claimed that acoustic chaos was obtained experimentally by the nonlinear interaction of two acoustic waves in a duct. In this comment a simple experimental setup and an analytical model is used to show that the dynamics of such systems corresponds to a quasiperiodic motion, and not to a chaotic one.
 [DOI: 10.1121/1.2978081]

PACS number(s): 43.25.Ts, 43.25.Zx, 43.58.Wc [ROC] Pages: 2702–2705

I. INTRODUCTION

The study of nonlinear systems has attracted the attention of scientists since the experiments of Jacques Hadamard were carried out in the early twentieth century. Since then, this phenomenon has been extensively studied by many authors, among others Poincairé, Kolmogorov, and Lorentz; a compilation of these and many other studies can be found elsewhere.[1] At present, the mathematical tools designed to analyze nonlinear dynamics and chaos are being applied to a great variety of dynamical systems; an example of this is the paper by Dong,[2] where an experimental setup consisting of two independently driven speakers placed at one end and on a side of a cylindrical duct and a microphone at the other end of the duct. The main objective of this paper was to define whether the resulting behavior of the interaction of two acoustic waves in a duct is chaotic or not. After analyzing an experimentally acquired time series and finding that (i) the largest Lyapunov exponent of the signal is positive, and (ii) the dimension of the reconstructed attractor is fractal, the authors concluded that the interaction of the two acoustic waves inside the duct is chaotic. In this comment, it is shown that the analysis presented in Dong's paper[2] is not sufficient to characterize the dynamics of such a system, since a quasiperiodic signal can be easily confused as a chaotic one if (a) the Lyapunov exponent analysis is not performed over an adequate time domain and (b) the error bars of the computed attractor fractal dimension include an integer value. Experimental data and a simple numerically generated signal are presented to support these affirmations.

Quasiperiodic behaviors are not difficult to produce or scarce in nature, but are difficult to characterize. In a simple case, a quasiperiodic motion can be thought of as a mixture of periodic motions with different incommensurable frequencies. As a result, a Fourier spectrum of a quasiperiodic signal consists of peaks at the values of the main fundamental frequencies and their linear combinations. The geometrical shape of its attractor in the phase space is a n-dimensional toroidal surface specified by the n fundamental frequencies. In fact, most of the nonlinear mathematical tools provide information to detect a quasiperiodic behavior. Detailed definitions and examples of Lyapunov exponents' analysis were given by Kantz,[3] but briefly, the Lyapunov exponent of a signal measures the rate of separation between nearby trajectories. If the distance between these trajectories does not change, it can be said that the dynamic response of the system is periodic and the largest Lyapunov exponent is zero. In contrast, if these trajectories diverge in time, exponentially to be precise, it could indicate that the system is chaotic and the largest Lyapunov exponent is positive. In a third case, if the system is quasiperiodic, these trajectories follow different paths but they would eventually approach each other only to diverge again and to repeat the process again and again. As a consequence, when a quasiperiodic behavior is suspected, the Lyapunov exponent analysis is required to be performed during times sufficiently large to identify these turn backs. In addition to this criterion, the time delay (denoted by J in Dong's paper[2]) used to calculate both the Lyapunov exponent and the dimension of the attractor D_c is also a critical variable to identify quasi-periodic behaviors. If it is taken too small, there would be almost no difference between the different elements of the delay vectors and this redundancy makes the vectors meaningless if the data is noisy and the

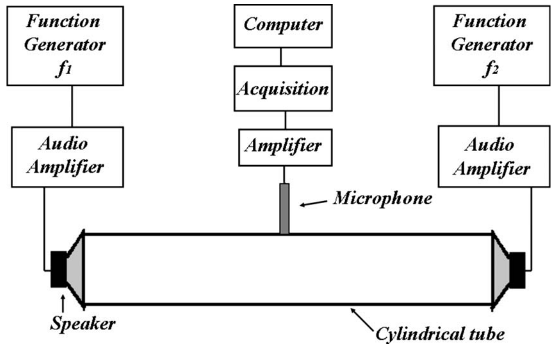

FIG. 1. Experimental arrangement.

variation of the signal during the lag J is less than the noise level.[4] On the other hand, if J is taken too large, then the different coordinates may be almost uncorrelated and the reconstructed attractor may become very complex, even if the "true" underlying attractor is a simple one. According to Swinney,[5] the method that estimates the value of J, using the first minimum of the time delayed mutual information is better and easier than the one that uses the autocorrelation.

In the next section, a simple experimental setup consisting of the same elements used in Dong's paper,[2] and which provides the time series analyzed below is presented. As expected, this data showed a Fourier spectrum with only the linear combination of the fundamental frequencies. Lyapunov exponents, return maps, and embedding dimensions were calculated for this data depicting a quasiperiodic motion and not a chaotic one. In addition, a simple quasiperiodic signal was created to test our dynamic-analysis battery; both, experimental and numerical results agree.

II. EXPERIMENTAL SETUP

As mentioned before, a simple experimental device, which basically consists of the same elements presented in Dong's paper,[2] was constructed and tested to prove our affirmations. Briefly, this setup consists of a cylindrical duct with speakers on both ends. The duct is a circular cylinder made of PVC, 60 cm long, 8 cm in diameter, and a 3 mm-wide wall. Two speakers PROAM model SPK-350 of 8 Ω and 75 W were placed at each one of the two ends of the duct. For each speaker, a function generator SRS was connected to

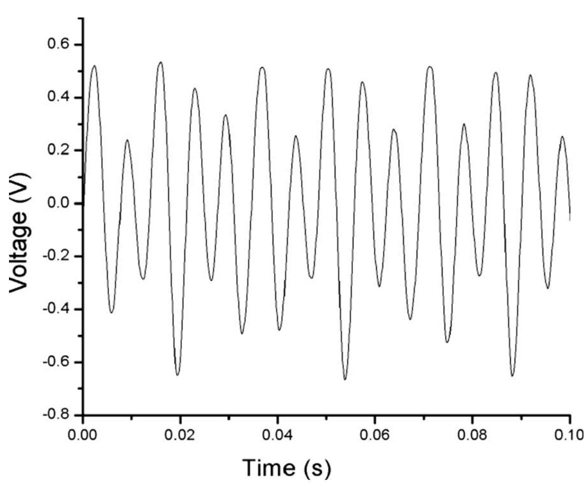

FIG. 2. Temporal signal with f_1=88, f_2=145 Hz, and V_{in}=2 V.

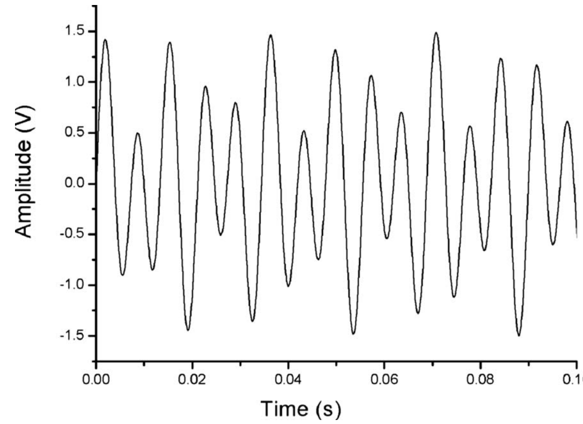

FIG. 3. Numerical signal with f_1=88 and f_2=145 Hz.

an audio amplifier (ONKYO, with total harmonic distortions <0.001) to produce the input-speaker voltage signals. A Bruel & Kjaer microphone with its Measuring Amplifier (2610 type) was installed on the side of the duct to detect the pressure signal. The output of the microphone amplifier was measured and recorded by an acquisition system HP 3206A connected to a computer using HP-VEE software.

Each speaker was driven by a pure sinusoidal signal of one of the two incommensurable frequencies: f_1=88 or f_2 =145 Hz, both of them with an amplitude of V_{in}=2 V (Fig. 1). A typical signal from the microphone is shown in Fig. 2; Fig. 3 shows a numerical signal used to mimic this dynamics which is simply a linear combination of two pure sinusoidals with frequencies f_1 and f_2.

III. ANALYSIS AND RESULTS

There are many criteria to classify dynamical systems, the most common are: the return map (commonly known as the experimental Poincaré section), Fourier spectrum, the phase space embedding dimension, the sign of the largest Lyapunov exponent, and the time delayed reconstructed attractor and its dimension.[3,6–8] Our analyzes were performed in terms of these time series analysis using the algorithms provided by Kantz.[8]

The step was to find out if the temporal signal was periodic or not via the return map (constructed by plotting the local extremal values of the time series as a function of the immediately preceding extrema). The return map can also help in the distinction between periodic and chaotic dynamics; it is, however, a qualitative test only and more careful analysis should be performed before trying to draw any conclusion. Qualitatively, there are several possibilities. The plot may consist of single points (whose number indicates the periodicity of the system) or it may be a simple (thin) closed curve that indicates a probable quasiperiodic dynamics. On the other hand, if the plot is an open (usually thick) curve, then the chances of having a chaotic dynamical system increase.

The return maps obtained for the signals are shown in Figs. 4 and 5, and correspond to a closed (period-one) curve. As mentioned above, this is a distinctive signature of a quasiperiodic motion.

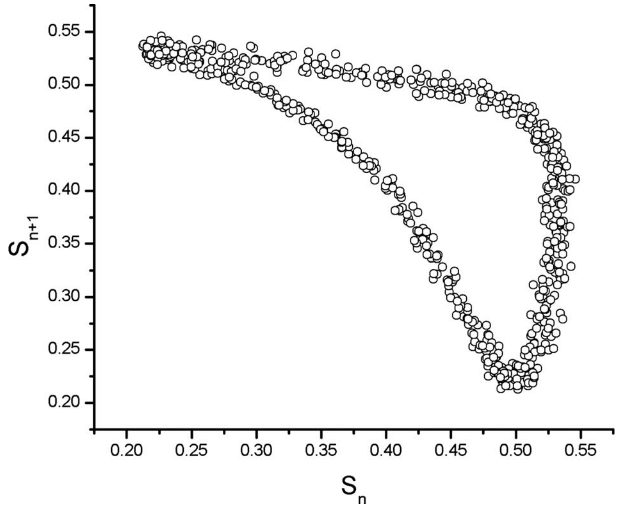

FIG. 4. Return map of the temporal signal in Fig. 2.

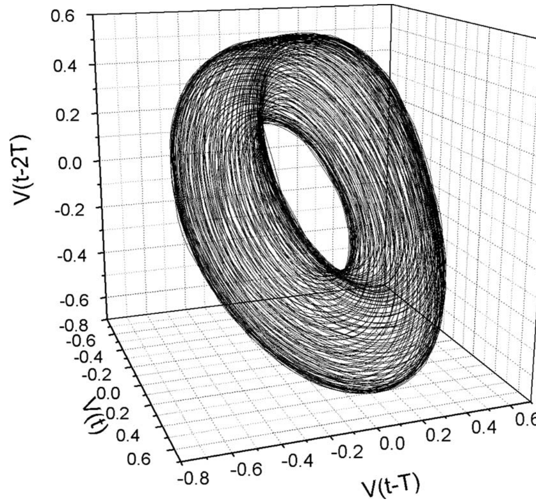

FIG. 6. Reconstructed attractor of the temporal signal in Fig. 2, and J $=2.1$ ms, $D_e=3$, and $D_c=2.02$.

To find the attractor we reconstructed the state space of the system using the well known method of time-delayed graphs. There are two important factors for this process: the time delay, J, used for the reconstruction and the embedding dimension, D, of the phase space where the attractor will be reconstructed. An incorrect choice of the embedding dimension for the reconstruction of the attractor could imply a wrong interpretation of the behavior of the dynamical system. An underestimate of the true dimension could show self-intersections or false periodicity due to the projection of a higher dimensional object. For the calculation of the time delay, an algorithm that uses the statistical ideas of the average mutual information was employed; as presented by Kantz.[8] An optimal value of $J=2.1$ ms is obtained using this method. To compute a good estimate of the embedding dimension, D_e, we used the false neighbors method which measures the Euclidean distance between two neighboring points of the attractor assuming it is embedded in a D-dimensional space and compares this value to the one obtained assuming a $(D+1)$-dimensional embedding space. If going from dimension D to $D+1$ unprojects away the two points by showing a clear increment of the Euclidean distance, this then shows that they were false neighbors. The dimension that has the lowest percentage of false neighboring points gives a good estimate for the true embedding dimension. Again, we used the algorithm provided by Kantz.[8]

The reconstructed attractors, using the values obtained via the time series analysis, are presented in Figs. 6 and 7, and correspond to tori. Using the Grassberger–Procaccia algorithm, the dimension of the attractor (D_c) was calculated to be $D_c=2.02\pm 0.09$, a value which lies on a range where no conclusions can be drawn as it could indicate an integer dimension. In Dong's paper,[2] D_c varies from 1.98 to 2.39. As this range surrounds an integer value, it should not be used to conclude a chaotic dynamics. In fact, there is another point of concern in Dong's paper[2] as their text-reported values of D_c do not correspond with the linear fits shown in their graphs; Figs. 12 and 13 of Ref. 2 show values of $D_c^{J=4}$ $=2.35$ and $D_c^{J=10}=2.33$, respectively. However, later in the text it is stated that $D_c^{J=4}=2.15$ and $D_c^{J=10}=2.5$.

The Kantz's algorithm[3,8] was used to calculate the largest Lyapunov exponent. The value of this exponent is obtained by computing the slope of a well-defined monotoni-

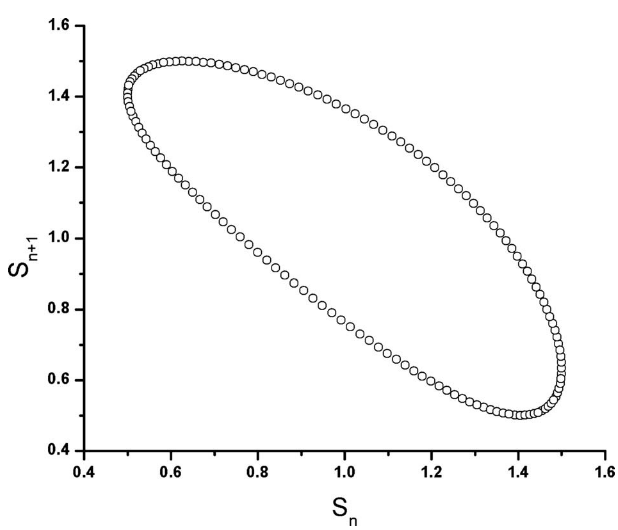

FIG. 5. Return map of the numerical signal in Fig. 3.

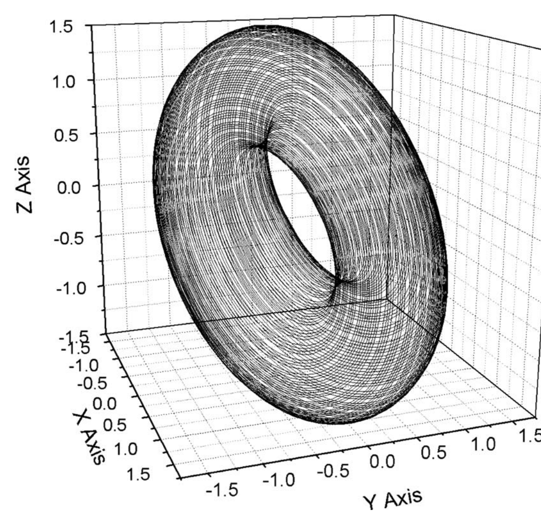

FIG. 7. Reconstructed attractor of the numerical signal in Fig. 3, and J $=2.1$ ms, $D_e=3$, and $D_c=1.95$.

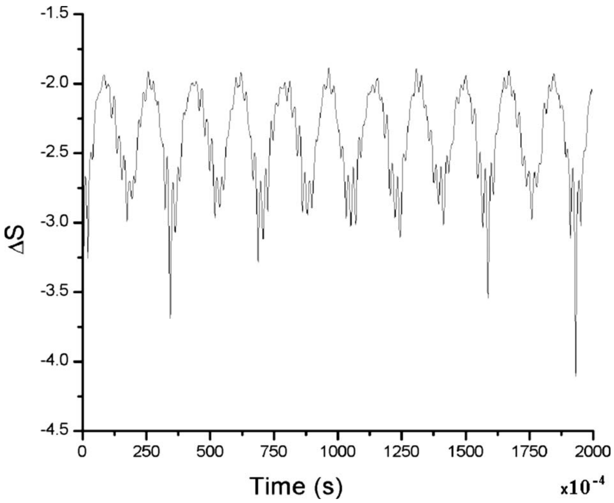

FIG. 8. ΔS for the experimental time series with $f_1=88$ Hz, $f_2=145$ Hz, $V_{in}=2$ V, $J=2.1$ ms, and $D_e=3$.

cally growing plot of the divergence of nearby trajectories in the phase space (in logarithmic scale because the algorithms assumed an exponential growth of this divergence). Our results are presented in Figs. 8 and 9. It is important to notice that the Lyapunov exponent (the slope of ΔS versus time), when calculated for reasonably large periods, is undefined. In particular, the divergence of adjacent trajectories in time, Figs. 8 and 9, is oscillatory for both the experimental and the numerical signals presented in this work (this periodic behavior is evident along the entire length of the data). This behavior is also observable in Figs. 10 and 11 in Dong's paper,[2] although there ΔS [ln(divergence)] is shown for a short period of time and only one oscillation is present. In addition, from the reconstructed attractor, the return map, the fractal dimension, and the Fourier spectrum of our experimental data we can conclude that there is not evidence of a chaotic behavior. First, the reconstructed attractor clearly represents a two-dimensional tori. Second, the return map is a closed and well defined curve. Third, the Fourier spectrum (not shown) peaks only at the original frequencies, their harmonics, and linear combinations of them. The lack of a continuous component in a frequency power spectrum is a hallmark of a periodic or quasiperiodic dynamics.

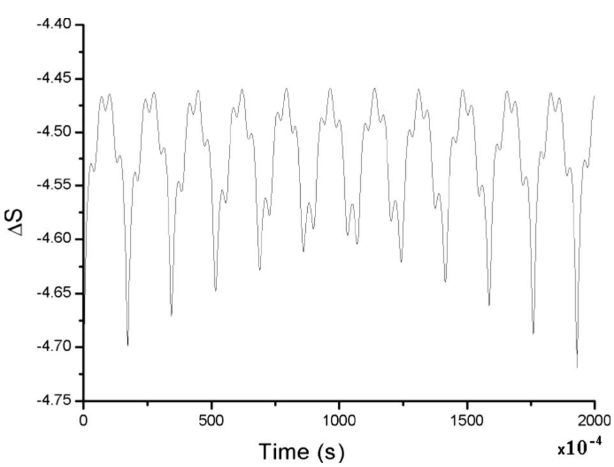

FIG. 9. ΔS for the numerical signal with $f_1=88$ Hz, $f_2=145$ Hz, $J=2.1$ ms, and $D_e=3$.

There is an artifact on the methods used to calculate the divergence of trajectories when they are applied to periodic (with more than one fundamental frequency) and quasiperiodic signals. Those methods are designed to look for an exponential divergence of nearby trajectories, and hence, the results are rather spurious when the divergence is not exponential. Interestingly, if one estimates the period of oscillation in graphs 10 and 11 in Dong's paper,[2] or Figs. 8 and 9 in this work, it turns out to be nothing else than the difference between the fundamental frequencies of the systems, i.e., 730 Hz in the original paper, and 57 Hz in our case (145−88 Hz). This tells us then, that if one thinks of two nearby trajectories in phase space, they never touch each other but get very close in regions near the "hole" of the torus and are relatively apart in the "opposite" regions of the same torus. This qualitatively explains the oscillatory shape of the plots presented in Figs. 8 and 9 in this work or in Figs. 10 and 11 of the original paper.

IV. CONCLUSIONS

From our analysis, we can conclude that the dynamics reported in Dong's paper[2] is quasiperiodic and not chaotic. This conclusion is based on the fact that the results in the calculations of the Lyapunov exponent were misinterpreted: incorrectly calculated to show an apparent positive value. This was presented as the strongest evidence for chaos. If the calculation would had been carried out for longer time span, it would had shown the oscillatory behavior in the divergence of the trajectories [ln(divergence) in Dong's paper,[2] or ΔS in this paper] from where the Lyapunov exponent is calculated, in the same way that it was presented here. To avoid this confusion, we propose the calculation of the largest Lyapunov exponent to be performed for longer times as this would allow the observation of a clear oscillatory behavior. Our experiments were also carried out all over the speaker optimum operational range, from 2 to 25 V, with similar results; no signatures of chaotic behavior were found.

ACKNOWLEDGMENTS

This work has been supported by CONACyT project 25116. A.A.C-P. thanks the UK Natural Environment Research Council for support.

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