Nasca lines: A mystery wrapped in an enigma

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We analyze the geometrical structure of the astonishing Nasca geoglyphs in terms of their fractal dimension with the idea of dating these manifestations of human cultural engagements in relation to one another. Our findings suggest that the first delineated images consist of straight, parallel lines and that having sophisticated their abilities, the Nasca artists moved on to the design of more complex structures. © *2003 American Institute of Physics.* [DOI: 10.1063/1.1587031]

Trying to gain knowledge from the diverse geometrical structure of the geoglyphs in the central Peruvian Andes, we have analyzed the biomorphic figures in terms of the dimension of each figure. A first finding shows that these glyphs have a noninteger (fractal) dimension and that its value allows a classification of the figures. This result and the reasonable hypothesis that the ability of the artists who created them increased with time, leads to a chronological ordering of the biomorphs that shows an interesting evolution from single straight lines to parallel connected lines and to curvilinear nonparallel drawings before designing whole plane trapezoids.

I. INTRODUCTION

The well-known Nasca geoglyphs in the central Andes of Perú (14°40′ to 14°55′ S latitude, 75°00′ to 75°10′ W longitude) have been one of the twentieth century rediscoveries that remain a mystery in many ways; our study of the lines that form these artistic figures shows that they may also be a riddle to modern mathematics. These geoglyphs were made by the pre-Inca inhabitants of the dry pampa by removing all the dark material (mainly stones) and revealing the clear granular substratum. Some indirect methods indicate that the geoglyphs were made before the year 600 (pottery shreds on the surfaces[1]), 525±80 (radiocarbon of a wooden post found nearby[2]), or from 190 BC to 660 (organic material "varnish" [3]); but due to their constitution, it is not possible to apply the usual direct dating methods and therefore, one is forced to look for alternative ways to estimate their age. The Nasca region has been subjected to immense perturbations (ignorant tourism mainly) and some of the geoglyphs are terribly damaged; although fortunately there are some good photographs that were taken when the figures had just been rediscovered, presumably by the mathematician Reiche,[2] who donated her lifelong work to humankind.

Research in various and diverse fields has shown that there is a tendency for some natural systems that evolve with time to increase their complexity;[4] animal physiology and behavior,[5] and human cultural manifestations not being the exception.[6,7] Having observed that there are different degrees of simplicity in the Nasca figures, even for the subset of biomorphs with deca- and hectometric dimensions, we have tried to establish their relative ages by measuring their fractal dimension. From our present day perspective, we conclude that simpler figures would have to be older than more complex ones, in agreement with the assumption of a tendency to greater complexity as their creators perfected their activity in time.

II. METHOD

Before measuring the fractal dimension of the available figures: three birds, two whales, a dog, a monkey, a spider, a pair of hands, and a tree (some shown in Fig. 1), we applied a careful digital process that involved intensity filtering, contrast magnification, and background-noise elimination, that resulted in a significantly increased contour-definition of the photographic images; two other methods for processing the figures were also employed to confirm consistent results.

The determination of the fractal dimension for each figure was carried out by applying the widely used box-counting method.[8,9] Briefly, the dimension of a black object over a white background or *vice versa*, and denoted by D, is defined as

$$D=\lim_{\varepsilon\to 0}\frac{\ln N(\varepsilon)}{\ln(1/\varepsilon)}, \qquad (1)$$

where $N(\varepsilon)$ is the number of boxes in a square grid of side-size ε required to cover the object in question. In our case, the side of the box was increased from one image pixel (0.32 mm) to the whole image size (72 dpi density resolution), and all images were previously scaled to a box 160 mm wide (the corresponding minimum height is 220 mm); this enabled us to avoid the loss of details when the small scales were analyzed. Figure 2 shows the fitting of two straight lines to the measurements obtained at different scales. As a uniform rule for avoiding the uncertainties due to upper and lower scale cut-offs, we discarded the three lowest and three highest points in each graph before performing the fitting; the linear correlation coefficients for the ten analyzed figures varied

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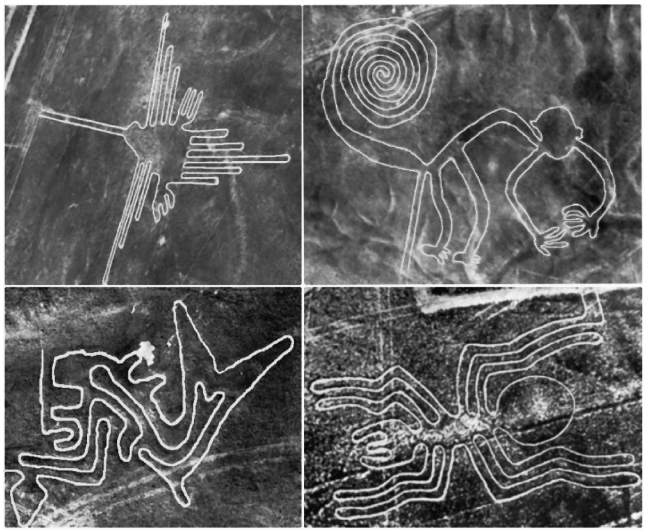

FIG. 1. Mosaic showing four of the analyzed biomorphs: humming bird, monkey, whale, and spider. The humming bird has a wingspan of 66 m, the actual size of the monkey is 135 m across, while the spider is 46 m long (Ref. 15).

from 0.995 to 0.999. Our results clearly show discernible differences in the values of the fractal dimension of distinct figures that imply an ordering of the biomorphs (Fig. 3). We will now use these differences and try to establish a chronological ordering of the figures in the following section.

III. RESULTS AND CONCLUSIONS

If one now adopts the point of view[10,11] that complexity and richness increases steadily from dimension one to dimension two, and the hypothesis of an increasing complexity in drawing as time evolves is accepted, one then has to place lines before trapezoids in time, i.e., lines older than plane figures (Fig. 3). This result is in agreement with some recent archeological findings,[12,13] and indicates that the Nasca art-

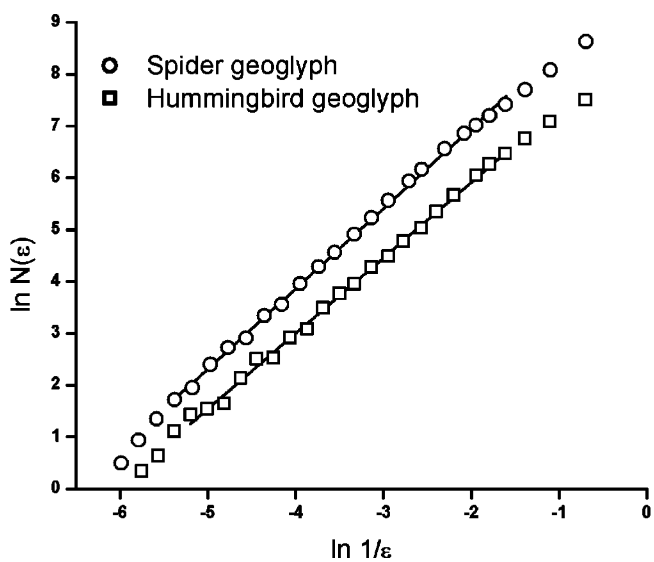

FIG. 2. Results of the box-counting method for two of the biomorphs in Fig. 1. The extension of the straight lines indicates the data used for the linear regression fittings.

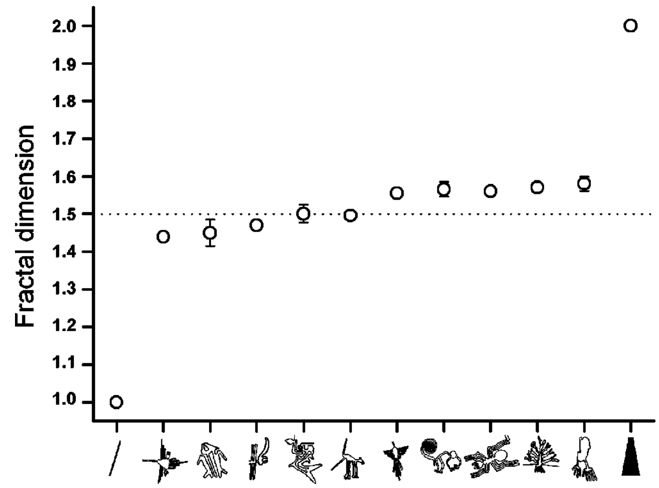

FIG. 3. Chronological order of the Nasca biomorphs according to their fractal dimension.

ists started their cultural activity with straight lines and, acquiring experience, moved on to the use of more complex geometries. The chronological order shown in Fig. 3 places the figures which essentially are combinations of straight, parallel lines before the ones that use mainly curves and nonparallel draws; the suggested relationship would then imply that the first figures are older and less sophisticated than the second set of figures, which look more complex in terms of the geometric elements just mentioned.

Although our study goes a different way when using the results—trying to establish a chronological ordering—it is similar to the analysis carried out for J. Pollock's paintings which enabled the establishing of an initial link between the visual preference or aesthetic quality and the value of the fractal dimension of an object; it has also opened a new field of research on the physiological effects of visual perception (see Ref. 14 for a review).

We do not expect our premises to be accepted as unquestionable but await for them to be contested and challenged; all we can ascertain at present, is that the Nasca imagery thus reinflames the debate on the meaning of art, a problem which may well continue as an elusive question even for modern mathematical tools.

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