Fractal dimension and self-similarity in *Asparagus plumosus*

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Abstract

We measured the fractal dimension of an African plant that is widely cultivated as ornamental, the *Asparagus plumosus*. This plant presents self-similarity, remarkable in at least two different scalings. In the following, we present the results obtained by analyzing this plant via the box counting method for three different scalings. We show in a quantitatively way that this species is a fractal.

1 Introduction

Nowadays it is frequent to use computational algorithms in order to produce images of plants and trees that resemble their natural counterparts. These visualizations, which present several symmetric bifurcations [1], encouraged us to analyze the *Asparagus plumosus* [2]. This plant is a native of Africa, but often cultivated in the rest of the world as an ornament. The plant can be easily identified: it is semi-climbing, has a typical height of 2 m, its main branches measure from 25 to 50 cm, and all branches have philiform divisions; its flowers are white and have six petals each, their fruits are purple spheres, 7 mm in diameter. Observed in some detail (Figs. 1-3), the 'leaves' of this plant, consist of repeated bifurcations from the main stem, showing a high degree of both, symmetry and scaling; these branching can also be observed even at the smallest scale. Two other peculiar characteristics of the 'leaves' of this plant are their flatness and their uniform green color. Although the branches may be dramatically different in shape (actually, Fig. 2 shows an atypical branch), we will show that their fractal dimension is the same.

2 Method

The method of box counting is widely known [3]. Briefly, the box counting technique consists of counting the number of boxes in a grid that intersect any part of an image that has been placed over it. In order to calculate the fractal dimension of the image, denoted by D, using a square grid of side size given by ε, one needs to analyze the changes in the number of boxes required to cover the image, N, as the size of the grid is reduced, *i. e.*,

$$D = \lim_{\varepsilon \to 0} \frac{\log N(\varepsilon)}{\log 1/\varepsilon}. \tag{1}$$

We have applied this method to the three branches shown in Figs. 1-3 at three different levels: the three different scales at which symmetry is observed. In Figs. 3-4, we visually exemplify the application of the box counting method. The 'leaves' we have designated as medium-size branches corre-

Fig. 1 A typical example of a main branch of *Asparagus plumosus*.

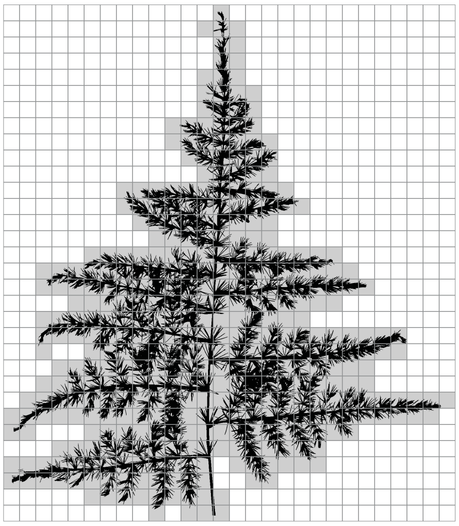

Fig. 3 Third example of a main branch; a grid of boxes with a side-length of 60 pixels is also shown (those boxes that have an intersection with the image are shaded in gray).

Fig. 2 An atypical main branch of *Asparagus plumosus*, note the differences in shape with respect to the usual branches in Figs. 1 and 3

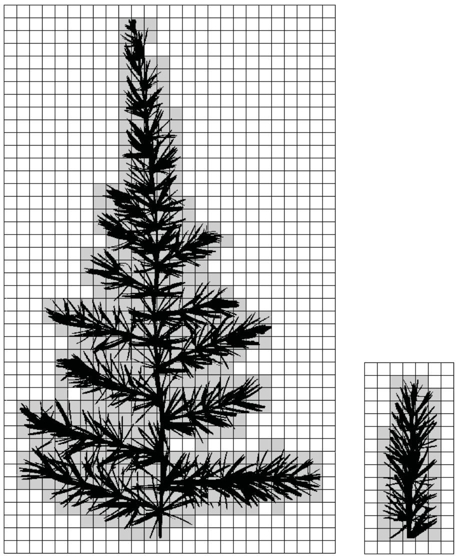

Fig. 4 Medium- and small-size branches lying on a square grid with side-length of 20 pixels.

spond to the ramifications at the lower right corner of the branches in Figs. 1-3, and those called small-size branches were selected from the medium-size ones following the same criteria; Fig. 4 exemplifies the selection for the main branch in Fig. 3. All the images were obtained by positioning the corresponding branch directly on a scanner (640 x 460 resolution, bit map images), and since the leaves are objects immersed in a two dimensional space, it was not necessary to use any kind of projection. The digital scanning was made in black & white, and in real scale. The side size of the square grid was varied from 1 to 200 pixels, by steps of 1 pixel. The original size of the main branch in Fig. 3 is 428.8 x 492.0 mm, 127.5 x 220.6 mm for the medium-size branch and 23.0 x 66.9 mm for the small one. Since our images all have well defined borders, there is no need to analyze the contour threshold [4].

3 Results

The values of N, obtained varying the grid size from 1 to 200 pixels, are shown in Figs. 5-7. This pixel range allows for a direct comparison in real scale of the results for the three levels at which similarity is observable. A bigger side-size box is not used because the width of the smallest branches (at the base) is 200 pixels, and therefore, a bigger side-size box would mean that a single box would almost cover the whole branch. Since the relations are linear over a wide range of ε values, the fractal dimension D is then given by the slope of the corresponding line, see Figs. 5-7. Finally, the values obtained for the fractal dimension of the three branches and at the three different scales, are shown in the Table together with the uncertainty in the slope (ΔD) and the correlation of the linear regression (R).

4 Conclusion

From the previous analysis, where we have shown that the fractal dimension of the three branches is practically the same, we can conclude that the shape of a branch of *Asparagus plumosus* is inde-

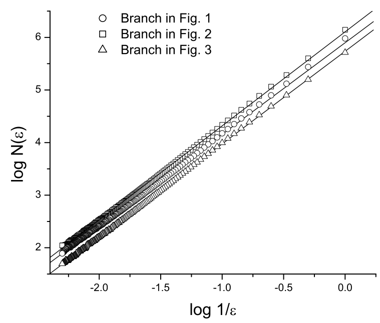

Fig. 5 Symbols represent the results of applying the box counting method to the main branches in Figs. 1-3. Straight lines show linear regressions performed for each data set.

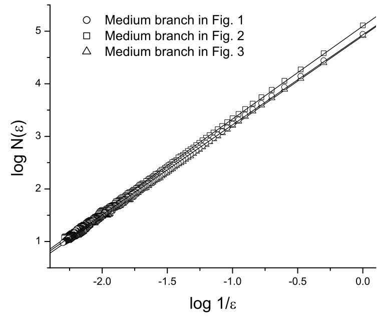

Fig. 6 Analogue of Fig. 5 for medium-size branches.

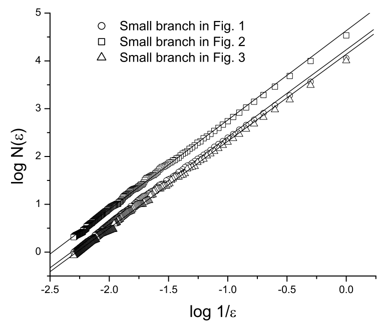

Fig. 7 Analogue of Fig. 5 for small-size branches.

pendent for the determination of its fractal dimension. The very small uncertainties in these values ($\Delta D/D < 3 \text{ x } 10^{-3}$) can be easily interpreted in terms of the high linear correlations shown in the Table. Accordingly, we can confirm the fractal dimension in this species, a new type of natural fractal being added to the extensive already well know gallery (for a recent, man-made example, see [5]). Additionally, since the value of the fractal dimension obtained from the analysis of the two bigger scales is indeed very similar, we can conclude that there is the same level of complexity at these two scales: the plant is self-similar. Unfortunately, we do not seem to find the same self-similarity at the smallest scale.

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Table

	Fractal dimension (D)			Uncertainty (ΔD)			Linear regression (R)		
Branch	Main	Med	Small	Main	Med	Small	Main	Med	Small
Fig. 1	1.742	1.712	1.825	0.003	0.003	0.005	0.999	0.999	0.999
Fig. 2	1.787	1.765	1.869	0.002	0.002	0.005	0.999	0.999	0.999
Fig. 3	1.760	1.722	1.819	0.002	0.003	0.006	0.999	0.999	0.998

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