Non-Isolated Singularities and derived Geometry

A celebration of the 60th birthday of David Massey.

July 29 - August 03, 2019, Cuernavaca, México

Book of Abstracts

Speakers

1 - Roberto Bedregal, Federal University of Paraiba.
2 - Daniel Cohen, Louisiana State University.
3 - James Damon, University of North Carolina.
4 - Christophe Eyral, Polish Academy of Sciences.
5 - Javier Fernández de Bobadilla, Ikerbasque-BCAM.
6 - Terrence Gaffney, Northeastern University.
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10 - Brian Hepler, Northeastern University.
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12 - Laurentiu Maxim, University of Wisconsin-Madison.
13 - Rodrigo Mendes - University of International Integration of Afro-Brazilian Lusophony
14 - Aurélio Menegon Neto - Federal University of Paraíba
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17 - Mutsuo Oka, Tokyo University of Science.
18 - Antoni Rangachev - University of Chicago.
19 - Edson Sampaio, Federal University of Ceará and BCAM - Basque Center for Applied Mathematics.
20 - Jörg Schürmann, University of Muenster.
21 - Jawad Snoussi, National Autonomous University of Mexico.
22 - Alexandru Suciu, Northeastern University.
23 - Kyoshi Takeuchi, University of Tsukuba.
24 - Michel Vaquie, Institut de Mathématiques de Toulouse.
25 - Matthias Zach, Leibniz Universitaet Hannover.

Titles and abstracts

1 - Roberto Bedregal

On Chern classes of singular varieties.

Chern classes had played an important role in Complex Geometry and Algebraic Geometry ever since its introduction by S-S Chern in 1946. Chern classes are characteristic classes associated to smooth complex manifolds or smooth algebraic varieties.In this talk I will address the two most important generalization which have appeared in the literature for singular varieties, the so called Fulton-Johnson and Schwartz-MacPherson classes. The difference between those two classes is the so called Milnor class. In this talk I will be mostly concerned with this Milnor class, which is a class supported on the singular locus of the variety.

2 - Daniel Cohen

Pure braid groups and direct products of free groups

I will discuss some properties and invariants of "arrangement groups" - fundamental groups of complements of complex hyperplane arrangements - largely in the context of the two classes of groups in the title.

3 - James Damon

Detecting the Characteristic Cohomology for Matrix Singularities

For a germ of a variety $\mathcal{V}, 0 \subset \mathbb{C}^N, 0$, a singularity $\mathcal{V}_0$ of "type $\mathcal{V}$" is given by a germ $f_0 : \mathbb{C}^n, 0 \to \mathbb{C}^N, 0$, which is transverse to $\mathcal{V}$ in an appropriate sense so that $\mathcal{V}_0 = f_0^{-1}(\mathcal{V})$. If $\mathcal{V}$ is a hypersurface germ, then so is $\mathcal{V}_0$, and by transversality $codim_{\mathbb{C}} sing(\mathcal{V}_0) = codim_{\mathbb{C}} sing(\mathcal{V})$ provided $n > codim_{\mathbb{C}} sing(\mathcal{V})$. So $\mathcal{V}_0, 0$ will exhibit singularities of $\mathcal{V}_0$ up to codimension n. Thus, non-trivial cohomology of the Milnor fiber of $\mathcal{V}_0$ may occur in dimensions between $n - 1 - dim_{\mathbb{C}} sing(\mathcal{V})$ and $n - 1$.

We better understand such a more complicated cohomology structure using the representation of $\mathcal{V}_0$ as a singularity of type $\mathcal{V}$. First, $\mathcal{V}_0$ contains a "characteristic subalgebra" obtained as the image of the cohomology of the Milnor fiber of $\mathcal{V}$. The cohomology of the Milnor fiber of $\mathcal{V}_0$ is then a graded module over $\mathcal{A}^*_{\mathcal{V}}(f_0; R)$. This module and algebra structure is an invariant, up to isomorphism, of the equivalence classes of such $\mathcal{V}_0$ preserving the defining equation of $\mathcal{V}$. Thus, part of the goal of understanding the cohomology of the Milnor fiber of $\mathcal{V}_0$ is to determine $\mathcal{A}^*_{\mathcal{V}}(f_0; R)$.

We consider the case where V denotes any of the varieties of singular $m \times m$ complex matrices which may be either general, symmetric or skew-symmetric (with m even). For these varieties we have shown that the Milnor fibers had compact "model submanifolds" for their homotopy types which are classical symmetric spaces in the sense of Cartan. As a result, it is shown that in each case the characteristic subalgebra is the image of specific exterior algebra (or in one case a module on two generators over an exterior algebra).

We introduce a criterion that detects a significant portion of the characteristic subalgebra by exhibiting a specific exterior subalgebra on l generators which it contains. The criterion is geometric and given in terms of the image of the defining germ containing a special type of subspace of "size" l which detects such (co)homology.

4 - Christophe Eyral

Lê numbers and Newton diagram

We give an algorithm to compute the Lê numbers of (the germ of) a Newton non-degenerate complex analytic function $f : (\mathbb{C}^n, 0) \to (\mathbb{C}, 0)$ in terms of certain invariants attached to the Newton diagram of the function $f + z_1^{a_1} + \cdots + z_d^{a_d}$, where d is the dimension of the critical locus of f and $a_1, \cdots, a_d$ are sufficiently large integers. This is a version for non-isolated singularities of a famous theorem of A. G. Kouchnirenko. As a corollary, we obtain that Newton non-degenerate functions with the same Newton diagram have the same Lê numbers.

5 - Javier Fernández de Bobadilla

Reflexive modules on normal Gorenstein Stein surfaces, their deformations and moduli
(Joint work with A. Romano Velazquez)

We generalize Artin-Verdier, Esnault and Wunram construction of McKay correspondence to arbitrary Gorenstein surface singularities. The key idea is the definition and a systematic use of a degeneracy module, which is an enhancement of the first Chern class construction via a degeneracy locus. We study also deformation and moduli questions. Among our main result we quote: a full classification of special reflexive MCM modules on normal Gorenstein surface singularities in terms of divisorial valuations centered at the singularity, a first Chern class determination at an adequate resolution of singularities, construction of moduli spaces of special reflexive modules, a complete classification of Gorenstein normal surface singularities in representation types, an study on the deformation theory of MCM modules and its interaction with their pullbacks at resolutions. For the proof of these theorems we prove several isomorphisms between different deformation functors that we expect that will be useful in further work.

6 - Terrence Gaffney

Whitney Equisingularity of Families of Non-isolated Singularities: The Past and the Future

Given a family of non-isolated singularities when is the family Whitney equisingular? The ideal answer would be to give a collection of invariants whose independence of parameter would give a necessary and sufficient condition for Whitney equisingularity to hold. This talk will look at some cases, among them essentially isolated determinantal singularities and hypersurface singularities, where this answer has been found. The pitfalls overcome suggest how to make future progress, and some results implementing this program will be described.

7 - Victor Goryunov

Vanishing cycles of matrix singularities

The talk is about holomorphic map germs $M : (\mathbb{C}^2, 0) \to Mat_n$, where the target is the space of either square, or symmetric, or skew-symmetric $n \times n$ matrices. The target contains the set Δ of all degenerate matrices, and our main object will be the vanishing topology of $M^{-1}(\Delta)$. Our attention is on the *singular Milnor fibre* of M, that is, the local inverse image V of Δ under a generic small perturbation of M. The variety V is highly singular, but, according to Lê Dũng Tráng's theorem, it is homotopic to a wedge of $(s-1)$-dimensional spheres.

The talk will start with introduction of local models for the spheres vanishing in the matrix context.

We will then prove the $\mu = \tau$ conjecture formulated by Damon for corank 1 map-germs M with a generic linear part, and a generalisation of this conjecture to the matrix version of boundary function singularities.

Bifurcation diagrams of matrix singularities will also be discussed, and a rather unexpected appearance of the discriminants of certain Shephard-Todd groups as such diagrams will be highlighted.

If time permits, possible approaches to the study of the monodromy will be mentioned.

8 - Xavier Gomez-Mont

TBA

9 - Nivaldo de Goes Grulha Junior

Generalizations of the local Euler obstruction and funcions with non-isolated critical set

The local Euler obstruction was defined by MacPherson [6] as a tool to prove the conjecture about existence and unicity of the Chern classes in the singular case. An equivalent definition was given in [3] by Brasselet and Schwartz, using stratified vector fields.

The local Euler obstruction was deeply investigated by many authors as Brasselet, Schwartz, Sebastiani, Lê, Teissier, Sabbah, Dubson, Kato and others. For instance, Lê D.T. and B. Teissier provide formula in terms of polar multiplicities [5]. In [1], Brasselet, Lê and Seade give a Lefschetz type formula for the local Euler obstruction. The formula shows that the local Euler obstruction, as a constructible function, satisfies the Euler condition relatively to generic linear forms.

In the paper [2], by Brasselet, Massey, Parameswaran and Seade, the authors study the obstacle for the local Euler obstruction to satisfy the Euler condition relatively to analytic functions with an isolated singularity at the considered point. That is the role of the so-called local Euler obstruction of f, denoted by $\mathrm{Eu}_{f,X}(0)$.

In joint work with Dutertre [4], base on the works [2, 7], we continue the study of the topological properties of functions defined on analytic complex varieties and we improve some results working with a germ-function f possibly with non-isolated singular set.

More recently, in her thesis (in finalization) Santana presented formulas to study the topology for the study of the pair of function-germs $f : (X, 0) \to (\mathbb{C}, 0)$ and $g : (X, 0) \to (\mathbb{C}, 0)$ in the case where g has a one-dimensional critical locus and give applications when f has isolated singularities and when it is a generic linear form.

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10 - Brian Hepler

Parameterized Surfaces and Hodge Theory

Parameterized spaces are a class of reduced complex analytic spaces with codimension-one singularities on which the (shifted) constant sheaf is perverse, and thus underlies a mixed Hodge module. For parameterized surfaces $V(f)$ in 3-space, we determine the weight filtration on the constant sheaf on $V(f)$, and in the special cases where $V(f)$ is an affine toric surface or the image of a finitely-determined map $\mathbb{C}^2 \to \mathbb{C}^3$, we also determine the Hodge filtration on the constant sheaf. This is joint work with Avi Steiner.

11 - Anatoly Libgober

Asymptotic of invariants of quasi-projective fundamental groups

We discuss distribution of invariants of fundamental groups of smooth quasi-projective varieties with simply connected compactification. One of invariants is the maximal rank of the free quotient and another is the rank of abelianization of the commutator. The discussion will include several boundness results as well as some conjectures.

12 - Laurentiu Maxim

Euclidean distance degree and the multiview conjecture

The Euclidean distance degree of an algebraic variety is a well-studied topic in applied algebra and geometry, with direct applications in geometric modelling, computer vision, and statistics. I will first describe a new topological interpretation of the Euclidean distance degree of an affine variety in terms of weighted Euler characteristics. As a concrete application, I will present a solution to the open problem in computer vision of determining the Euclidean distance degree of the affine multi view variety. Secondly, I will present a solution to a conjecture of Aluffi-Harris concerning the Euclidean distance degree of projective varieties. Projective varieties appear naturally in low rank matrix approximation, formation shape control, and all across algebraic statistics. (Joint work with J. Rodriguez and B. Wang.)

13 - Rodrigo Mendes

Link of Lipschitz Normally embedded sets

A semialgebraic germ $(X, x_0) \subset (\mathbb{R}^n, x_0)$ has two natural metrics: The outer (or euclidian) metric and the inner metric (or length metric). When the two metrics are bi-Lipschitz equivalent and the bi-Lipschitz homeomorphism is given by the identity map, we say that the germ (X, x_0) is Lipschitz normally embedded. This notion is in a recent development and enables one to understand the nature of the singular types of algebraic varieties on the metric point of view. In this talk, we prove that a germ (X, x_0) is Lipschitz normally embedded if, and only if, the family of ϵ-Links (for small ϵ) $\{X \cap \mathbb{S}^{n-1}(x_0, \epsilon)\}_\epsilon$ is Lipschitz normally embedded with uniform constant. In the last part, we will discuss about the behaviour of the some metric invariants of singular germ using this equivalence. This a joint work with Edson Sampaio.

14 - Aurélio Menegon Neto

TBA

15 - Orlando Neto

Limits of Tangents of Surfaces

We compute the limit of tangents of the germ of a surface. We obtain as a byproduct an embedded version of Jung's desingularization theorem for surface singularities with finite limits of tangents.

16 - Juan José Nuño Ballesteros

Whitney equisingularity of corank 1 map germs from $\mathbb{C}^n$ to $\mathbb{C}^{n+1}$

Let $f : (\mathbb{C}^n, S) \to (\mathbb{C}^{n+1}, 0)$ be a map-germ which has finite $\mathcal{A}$-codimension, or equivalently, which has isolated instability. The main invariant of f is the image Milnor number $\mu_I(f)$, introduced by D. Mond. By a result of Lê, the image of a stabilization of f has the homotopy type of a bouquet of spheres and $\mu_I(f)$ is defined as the number of such spheres, in analogy with the classical Milnor number of a hypersurface with isolated singularity.

We will restrict ourselves to the corank 1 case and prove that $\mu_I(f) = 0$ if and only if f is stable. This is what we call the "weak Mond's conjecture", for its relationship with the Mond's conjecture which says that $\mu_I(f)$ is greater or equal to the $\mathcal{A}_e$-codimension of f, with equality if f is weighted homogeneous. We will use the "weak Mond's conjecture" to prove that if a 1-parameter family f_t is μ_I-constant, then the family f_t is excellent in the sense of Gaffney. This gives a positive answer to a conjecture by K. Houston.

We also will introduce a new invariant $\mu_D(f)$ which we call the double point Milnor number. This is equal to the number of spheres in the double point locus in the source of a stabilization of f. Our main result is that a 1-parameter family f_t is Whitney equisingular if and only if the sequences μ_I^* and μ_D^* are constant in the family.

This is a joint work with R. Gimenez-Conejero.

17 - Mutsuo Oka

On the Milnor fibration for $f(\mathbf{z})\bar{g}(\mathbf{z})$

We consider a mixed function of type $H(\mathbf{z}, \bar{\mathbf{z}}) = f(\mathbf{z})\bar{g}(\mathbf{z})$ where f and g are convenient holomorphic functions which have isolated critical points at the origin and we assume that the intersection $f = g = 0$ is a complete intersection variety with an isolated singularity at the origin and H satisfies the multiplicity condition. We will show that H satisfies Hamm-Lê condition. In particular, H has a Milnor fibration at the origin. We give examples which does not satisfy the Newton multiplicity condition where one does not have Milnor fibration and the other has Milnor fibration.

18 - Antoni Rangachev

TBA

19 - Edson Sampaio(1),(2)

The generalized Poincaré's Conjecture for links of LNE complex analytic sets

A set $X \subset \mathbb{R}^n$ is called Lipschitz normally embedded (LNE) if the identity map between X endowed with the inner distance and X endowed with the induced euclidean distance is a bi-Lipschitz homeomorphism. It is well known that if a closed (i.e., compact and without boundary) m-manifold M is homotopy equivalent to the m-sphere then M is homeomorphic to $\mathbb{S}^m$, before its proof this result was known as the generalized Poincaré's Conjecture. In this talk, we study the generalized Poincaré's Conjecture in the case of links of LNE complex analytic sets. In fact, if $X \subset \mathbb{C}^n$ is a LNE complex analytic set, $X_t := (\frac{1}{t}X) \cap \mathbb{S}^{2n-1}$ for $t > 0$ and $d = dimX$, we prove that the following statements are equivalent:

(1) X is a homology $2d$-manifold (i.e., $H_*(X; X \setminus \{x\}) = H_*(\mathbb{R}^{2d}; \mathbb{R}^{2d} \setminus \{0\})$ for all $x \in X$) and $\pi_1(X_\epsilon) = \{0\}$, where $\epsilon > 0$ is the Milnor's radius of X at 0;
(2) The limit (with respect the Hausdorff distance) $X_0 := H\ lim_{t \to 0+} X_t$ is homotopy equivalent to $\mathbb{S}^{2d-1}$;
(3) The tangent cone of X at 0 is a linear subspace of $\mathbb{C}^n$.
(4) X is smooth at 0.

As a consequence, we obtain that if Y is the link at 0 of a LNE complex analytic set X (not necessary with isolated singularity), X is a homology n-manifold and Y is homotopy equivalent to $\mathbb{S}^{2d-1}$ then Y is diffeormophic to $\mathbb{S}^{2d-1}$. In particular, if X is a LNE complex analytic set and also a topological manifold then X is smooth. Joint works with Alexandre Fernandes(1).

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(2) BCAM - Basque Center for Applied Mathematics, Bilbao, Basque Country - Spain.

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20 - Jörg Schürmann

(Degenerate) affine Hecke algebras and (motivic) Chern classes of Schubert cells

We explain in the context of complete flag varieties X=G/B the relation between (motivic) Chern classes of Schubert cells and convolution actions of (degenerate) affine Hecke-algebras as in the work of Ginzburg and Tanisaki. This is joint work with P. Aluffi, L. Mihalcea and C. Su.

21 - Jawad Snoussi

TBA

22 - Alexandru Suciu

Germs of representation varieties and cohomology jump loci

Given a finitely generated group π and a complex, linear algebraic group G, the representation variety $\mathrm{Hom}(\pi, G)$ has a natural filtration by the cohomology jump loci associated to a rational representation $G \to \mathrm{GL}(V)$. The infinitesimal counterpart of $\mathrm{Hom}(\pi, G)$ around the trivial representation is the space of $\mathfrak{g}$-valued flat connections on an appropriate commutative, differential graded algebra (A, d). This space admits a corresponding filtration by the resonance varieties associated to the tangential representation $\mathfrak{g} \to \mathfrak{gl}(V)$ of the Lie algebra of G. I will explain how one can understand the local behavior of all these varieties, at least in some favorable situations of geometric interest. This approach works particularly well in the case when $G = \mathrm{SL}(2, \mathbb{C})$ and π is either an Artin group or the fundamental group of a smooth, quasi-projective variety. This is joint work with the late Stefan Papadima.

23 - Kyoshi Takeuchi

On Irregularities of Fourier Transforms of Regular Holonomic D-Modules

We study Fourier transforms of regular holonomic $\mathcal{D}$-modules. By using the theory of Fourier-Sato transforms of enhanced ind-sheaves developed by Kashiwara-Schapira and D'Agnolo-Kashiwara, a formula for their enhanced solution complexes will be obtained. Moreover we show that some parts of their characteristic cycles and irregularities are expressed by the geometries of the original $\mathcal{D}$-modules. Joint work with Yohei Ito.

24 - Michel Vaquie

Introduction to Derived Algebraic Geometry

Abstract TBA

25 - Matthias Zach

TBA